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Application of Finite Element Method to Create Animated Simulation of Beam Analysis for the Course of Mechanics of Materials

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ABSTRACT: *"Mechanics of Materials" is a basic course for the students in department of mechanical engineering and civil engineering. The traditional teaching method in the class room is that teacher offers the students basic concepts, develops the formulas and let the students practice the exercises, or may add the experimental tests to help student to understand. Students have to try understanding the analysis and formula, and have to do many exercises to catch up the physical meaning of mechanics of materials. For increasing the student interests and making an impression on this course, the finite element method combined with the animated graphic technology is used to develop the virtual simulation of beam analysis. Beam is a major topic of "Mechanics of Materials", students will catch up the principles of mechanics if they completely understand beam behaviors through the beam analysis. We developed a series of animated pictures to describe the beam behaviors with different geometries, material properties, and loading conditions. Only the rectangular cross section is considered and the cantilever beam analysis is developed in this paper. Also the elastic material properties are used. Presentation of the animated figures in the course, we found student accept easily the concept of mechanics of materials, and have more confidences for the future study. They also like to learn more virtual simulation technology such as finite element method to increase their ability.*

1 INTRODUCTION

"Mechanics of Materials" is a basic and fundamental course for the undergraduate students in department of Mechanical Engineering and Civil Engineering. This course offers the students various engineering concepts of the strength and mechanics of materials. In many topics, deflection of beam is one important topic in this course. Under the action of applied forces the axis of a beam deflects from its initial position. Accurate values for beam deflection are sought in many practical cases. Elements of machines must be sufficiently rigid to prevent misalignment and to maintain dimensional accuracy under load. In buildings, the floor beams cannot deflect excessively to deviate the undesirable psychological effects on the occupants and to minimize to prevent distress in brittle finish materials. Likewise, in formation on deformation characteristics of members is essential in the study of vibrations of machines as well as of stationary and flight structures.

Basic differential equations for the deflection of beams will be described in this paper first. Next solution of these equations is illustrated in detail. Only deflections caused by forces acting perpendicularly to the axis of a beam are considered. The basic theory developed in this paper is limited to deflection which are small in relative to span length. Only deflections caused by bending are considered in this paper. Those due to shear are negligible.

Basic Assumptions

For the present it is assumed that only straight beams having constant cross-sectional areas with an axis of symmetry are to be included in the discussion. The most fundamental hypothesis of the flexure theory, based on the geometry of deflections, it may be stated thus:

1. Plane sections through a beam, taken normal to its axis, remain plane after the beam is subjected to bending. In a beam subjected to bending, strains in its fibers, very linearly or directly as their respective distance from the neutral surface.

2. Hooke's law is applicable to the individual fibers, i.e., stress is proportional to strain. The same elastic modulus E is assumed to apply to material in tension as well as compression. The Poisson's effect and the interference of the adjoining differently stresses fibers are ignored.

The governing differential equation for deflection of elastic beam

The differential relations among the applied loads, shear, and moment can be combined to yield the following useful segment of equations:

$$M = EI\frac{d^2v}{dx^2}$$

$$V = \frac{dM}{dx} = EI\frac{d^3v}{dx^3}$$

$$q = \frac{dV}{dx} = EI\frac{d^4v}{dx^4}$$

Where	v	– deflection of the elastic curve,
	M	– moment,
	V	– shear,
	Q	– the applied load,
	E	– Young's modulus,
	I	– moment of inertia.

The choice of equation for a given case depends on the case with which an expression for load, shear, or moment can be formulated. Fewer constants of integration are needed in the lower-order equations. For the solution of beam deflection problems, in addition to the differential equations, boundary conditions must be prescribed. Several types of homogeneous boundary conditions are as follows:

1. Clamped or fixed support: In this case the displacement v and the slope dv/dx must vanish. Hence at the end considered, where x=a, v(a)=0, $v''(a) = 0$.
2. Roller or pinned support: At the end considered, no deflection v nor moment M can exist. Hence v(a)=0, M(a)=EI$v''(a)$=0. Here the physically evident condition for M is related to the derivative of v with respect to x.
3. Free end: Such an end is free of moment and shear. Hence M(a)=EI$v''(a)$=0, V(a)=EI$v'''(a)$=0.
4. Guided support: In this case free vertical movement is permitted, but the rotation of the end is prevented. The support is not capable of resisting any shear. Therefore, $v'(a)$ =0, V(a)=EI$v'''(a)$=0.

From the above description, the governing equation of the beam deflection combined with different boundary conditions and loading conditions, many cases for the baem can be analyzed. If the cross section of the beam is considered, then more cases can be found out. Most of the time, students just spent much time on driving the equations and solve the problems to get the solutions, and forget the physical meaning in the solutions. For avoiding these defects, we will present a method that combined the solutions of the beam with the finite element method and using the graphical interface capability in the finite element code to show the physical phenomena of the beam deflection.

Procedures of the studies

For the beam deflection problems, the procedures for the study are listed in the following:

a. First the engineering problem for the beam deflection is described, then
b. The closed form solution is presented.
c. The finite element model is generated.
d. The results obtained from © and (d) are compared with each other.

e. The deflection of the beam is plotted for each case and differences between them are pointed out.
f. The animated picture is presented in the class and explain the boundary conditions, loading conditions effect on the beam deflection.
g. Evaluate the response of the student and make a suggestion for the next subject.

Example

A cantilever beam made by the aluminum alloy, the beam length is 100 m, the cross section that is plotted in Figure1 is 200 m^2, width is 20 m and thickness is 10 m. It was fixed at the left end, the applied load will depend on the conditions and listed in the following:

1. Case (a): loading is applied at the right end, and F=500 N shown in Figure 2.
2. Case (b): loading is applied at the point that located 60 m from the fixed end, F is also 500 N and plotted in Figure 3.
3. Case (c): loading is a moment and applied at the right end of the beam as shown in Figure 4 and moment is 500N-m.
4. Case (d); loading is a moment and is applied as shown in Figure 5. The moment is 500 N-m.

The material properties used here is E=80 Gpa and Poisson's ratio is 0.33. The closed form solution for case (a),(b), (c), and (d) are listed in the following:

Case a: $v = \dfrac{Px^2}{6EI}(3L - x)$ and $I = \dfrac{BH^3}{12}$

Case b: $v = \dfrac{Px^2}{6EI}(3a - x) \qquad (0 \le x \le a)$

$v = \dfrac{Px^2}{6EI}(3x - a) \qquad (a \le x \le L)$

Case c: $v = \dfrac{Px^2}{6EI}(3L - x)$

Case d: $v = \dfrac{M_0 x^2}{2EI} \qquad (0 \le x \le a)$

$v = \dfrac{M_0}{2EI}(2x - a) \qquad (a \le x \le L)$

(3) The finite element model is created by using ANSYS finite element software code. In this model, the element LINK1 is selected and plotted in Figure 6. Total number of elements is 10 elements and 11 nodes. The model can be plotted in 3-D model shown in Figure 7 and the cross section was displayed.

(4) The results calculated from the closed form solutions and FEA are listed in Table 1 to 4. They are also plotted in Figure 8 to 11. All of these results indicate FEA calculations can completely agree with those obtained from the closed form solutions.

(5) The maximum deflection for case (a) is plotted in Figure 12: for case (b) is plotted in Figure 13; for case (c) is presented in Figure 14; and for case (d) is shown in Figure 15. All of these deflections can be animated to become a movie that can be presented in the class. Due to a series of action pictures, student can easily check and observed the beam deflection under different boundary conditions and loading conditions.

CONCLUSIONS

Finding the deflection of the beam under different conditions is an important section in the course of "Mechanics of Materials." After developing this course combined the theoretical formula with finite element analysis can give the student strong impressions about the beam deflection. After students viewed the animated pictures of the beam deflection, they generally expressed they have more understandings about the loading conditions and boundary conditions effect on the beam deflection. In this course, student can learn basic theory of the beam and also can learn how to apply FEA to verify the closed form solutions. This may improve student's confidence in learning the other courses of Mechanics. After this development, we will follow the same procedures to expand this method to the other section of "Mechanics of Materials." It may also extend this method to the other mechanical courses such as Fluid Mechanics, Thermal Mechanics, and Structural Dynamics.

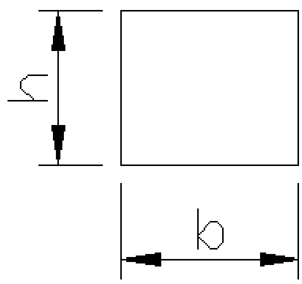


Figure 1: The cross section of the beam

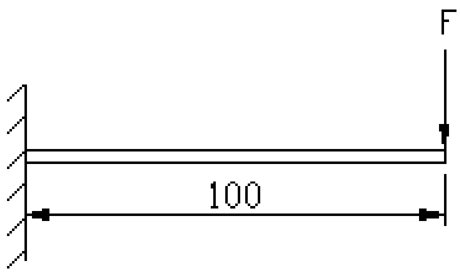


Figure 2: Load applied at the end of the beam for case (a)

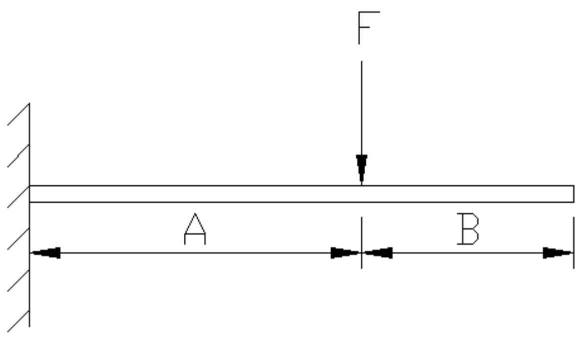


Figure 3: Load applied at somewhere of the beam for case (b)

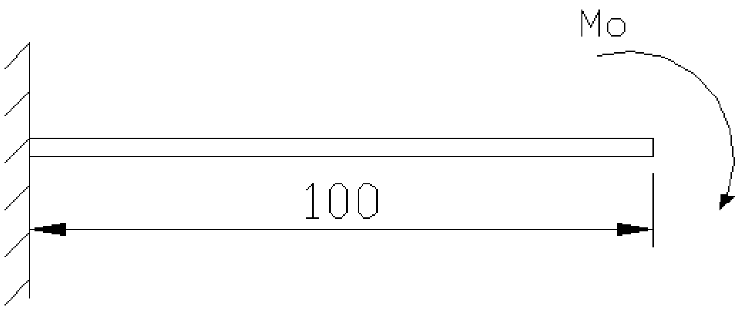


Figure 4: Moment applied at the end of the beam for case (c)

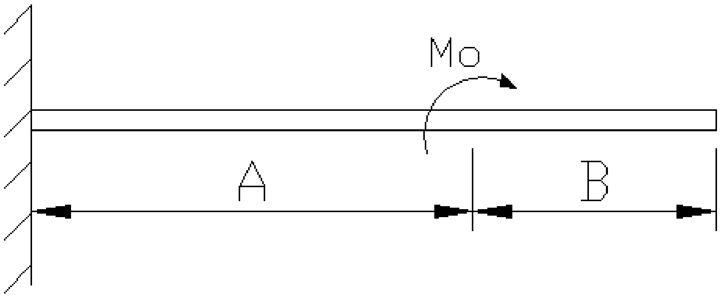


Figure 5: Moment applied at somewhere of the beam for case (d)

Table 1: Deflection from the theoretical solution and FEA for case (a)

	10(m)	20(m)	30(m)	40(m)	50(m)	60(m)	70(m)	80(m)	90(m)	100(m)
FEA	2.16E-08	7.95E-08	1.62E-07	2.71E-07	4.02E-07	5.52E-07	7.16E-07	8.92E-07	1.08E-06	1.26E-06
THEROY	1.81E-08	7.00E-08	1.52E-07	2.60E-07	3.91E-07	5.40E-07	7.04E-07	8.80E-07	1.06E-06	1.25E-06

Table 2: Deflection from the theoretical solution and FEA for case (b)

	10(m)	20(m)	30(m)	40(m)	50(m)	60(m)	70(m)	80(m)	90(m)	100(m)
FEA	1.16E-08	4.02E-08	8.73E-08	1.44E-07	2.08E-07	2.76E-07	3.43E-07	4.11E-07	4.78E-0	5.46E-07
THEROY	1.06E-08	4.00E-08	8.44E-08	1.40E-07	2.03E-07	2.70E-07	3.38E-07	4.05E-07	4.73E-07	5.40E-07

Table 3: Deflection from the theoretical solution and FEA for case (c)

	10(m)	20(m)	30(m)	40(m)	50(m)	60(m)	70(m)	80(m)	90(m)	100(m)
FEA	1.88E-10	7.50E-10	1.69E-09	3.00E-09	4.69E-09	6.75E-09	9.19E-09	1.20E-08	1.52E-08	1.88E-08
THEROY	1.88E-10	7.50E-10	1.69E-09	3.00E-09	4.69E-09	6.75E-09	9.19E-09	1.20E-08	1.52E-08	1.88E-08

Table 4: Deflection from the theoretical solution and FEA for case (d)

	10(m)	20(m)	30(m)	40(m)	50(m)	60(m)	70(m)	80(m)	90(m)	100(m)
FEA	1.88E-10	7.50E-10	1.69E-09	3.00E-09	4.69E-09	6.75E-09	9.00E-09	1.13E-08	1.35E-08	1.50E-08
THEROY	1.88E-10	7.50E-10	1.69E-09	3.00E-09	4.69E-09	6.75E-09	9.00E-09	1.13E-08	1.35E-08	1.50E-08

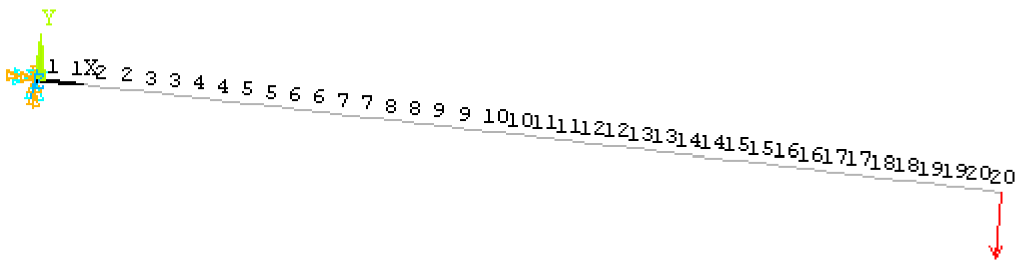


Figure 6: Finite element model for the beam

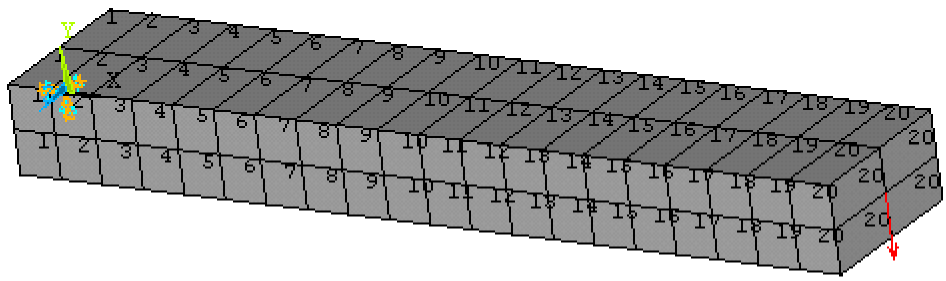
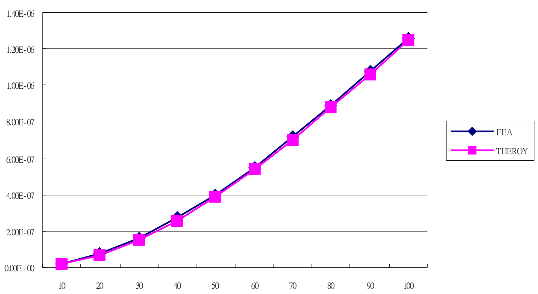
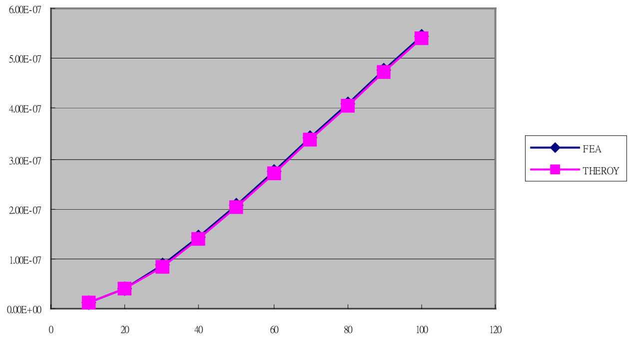


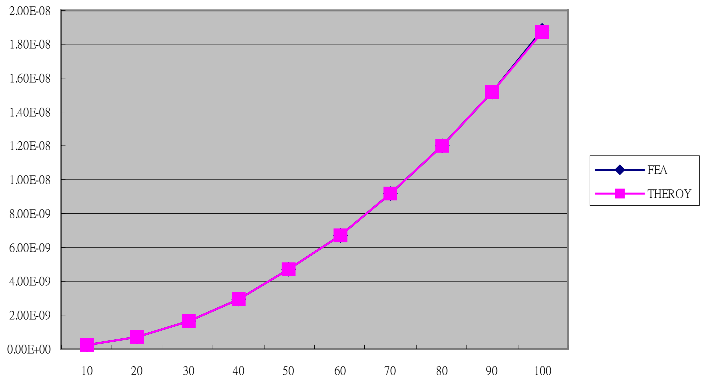
Figure 7: 3-D finite element model for the beam



Figuure 8: Results from the theoretical solution and FEA for case (a)



Figuure 9: Results from the theoretical solution and FEA for case (b)



Figuure 10: Results from the theoretical solution and FEA for case (c)

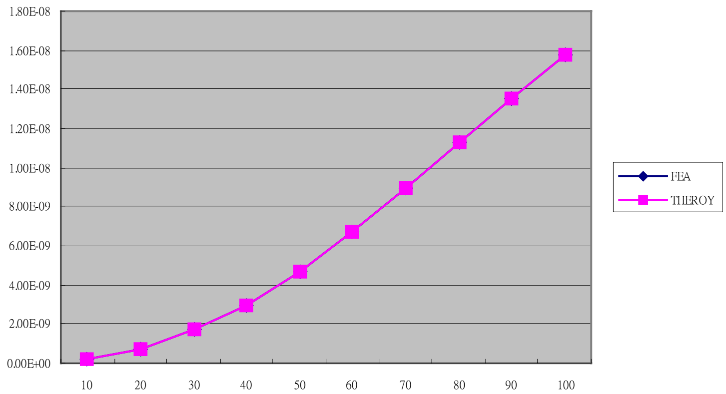


Figure 11: Results from the theoretical solution and FEA for case (d)

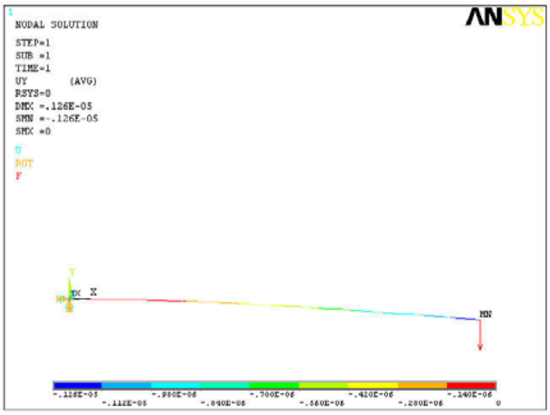


Figure 12:The deflection of the beam for case (a)

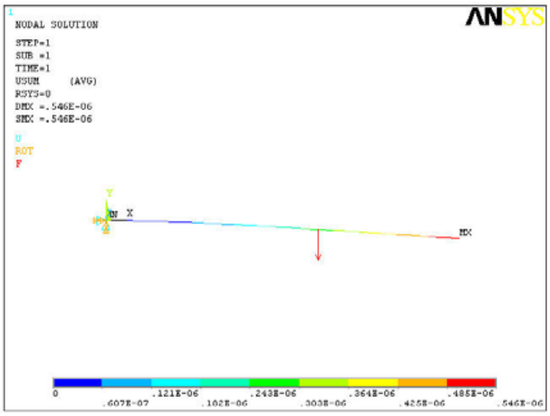


Figure 13: The deflection of the beam for case (b)

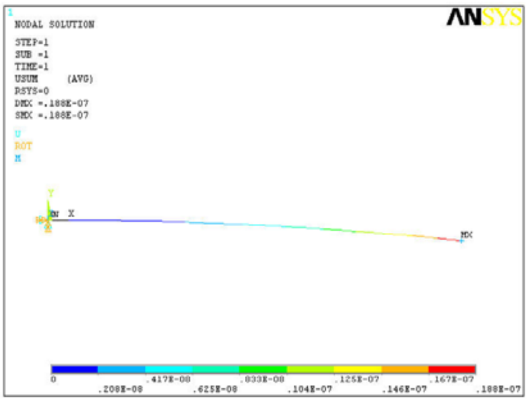


Figure 14: The deflection of the beam for case (c)

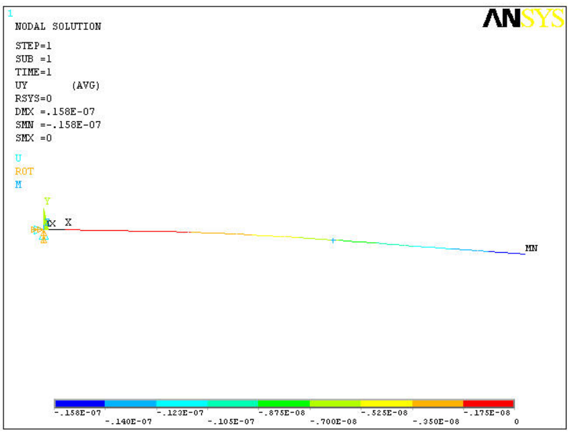


Figure 15: The deflection of the beam for case (d)