A Fuzzy Logic 2000 educational package for *Mathematica*®

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Abstract: This paper describes a software which is used in many universities around the world. Fuzzy logic is a rapidly expanding field in today's computer industry with new and exciting applications being developed all the time. Yet, even as the field has grown, some areas that need attention still remain. One of these areas is educational software. The goal of this project was to create a teaching aid to assist in learning the concepts of fuzzy logic and to help perform the basic, yet often tedious, calculations involved. Of special importance was the graphical representation needed to help the user visualize and better understand the operations in fuzzy logic. The Fuzzy Logic Package also needed to include the full range of fuzzy operators to insure completeness.

The software package described above was written as an extension to the program *Mathematica*®. *Mathematica*® by itself is a valuable tool that can be used for numerical calculations, graphical representation of functions, symbolic calculations, and programming. The new package is a set of functions that can be easily read in and used in conjunction with *Mathematica*®'s others capabilities. The project's initial goal was to implement a software to aid in study of fuzzy logic. This goal was achieved when the package was used as a study aid for fuzzy logic class in several universities around the world. However, the package has also shown that it can be used for many applications above and beyond the original intent. The ease with which digital fuzzy sets and relations can be entered and manipulated makes this package an invaluable tool for anyone interested in studying or applying the techniques of fuzzy logic in digital image processing, clustering or more advanced fuzzy modeling.

Keywords: fuzzy set theory, *Mathematica*®, Fuzzy Logic Package

1. Introduction

As fuzzy logic usage increases so does the number of people who wish to learn the concepts. To facilitate this learning, a software package was developed. This package contains a wide variety of operations and graphical capabilities which allow the user to explore fuzzy logic. The package was implemented as an extension to the program *Mathematica*®. *Mathematica*® is one of the best mathematical calculation and manipulation programs available today. It has found wide use in many scientific and educational areas and its popularity is still growing. The choice of implementing a fuzzy logic package as an extension to *Mathematica*® not only provides a rich supply of existing functionality, but also makes the package[10] more attractive to many potential users.

A fuzzy set **X** in the universal space **U** is defined by a function that returns values in the range **[0,1]**.

$$\mathbf{X} : \mathbf{U} \text{ -> } [0,\ 1]$$

A fuzzy set **X** is often written as a set of pairs **{u, X(u)}** [1,8], where **u** is an element in the universal space **U** and **X(u)** is the grade of membership function of element **u** in a fuzzy set **X**. Similarly, if **U** and **V** are two collections of objects, a fuzzy relation **R** in the Cartesian product **U x V** is a function that returns values from the interval **[0, 1]**.

$$\mathbf{R} : \mathbf{U} \times \mathbf{V} \text{ -> } [0,\ 1]$$

Financial support from the Undergraduate Research Opportunities Program of the University of Minnesota is gratefully acknowledged.

Similar to a fuzzy set a fuzzy relation can be written as an ordered set. A relation **R** is represented by a set of triplets **{u, v, R(u, v)}**, where **u** is an element of the universal space **U**, **v** is an element of the universal space **V**, and **R(u, v)** is the value of the function **R** at **(u, v)** [5]. There exist numerous operations that can be performed on fuzzy sets and fuzzy relations. Some of the basic ones being: max-union, min-intersection,standard complement. The fuzzy set operations are

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functions that map two fuzzy sets in the same universal space to a new fuzzy set in that same space. Fuzzy relation operations map two fuzzy relations in the same space to a new relation in that space. A fuzzy composition operation on relations **R : U x V** and **S : V x W** results in a new fuzzy relation in the universal space **U x W**. For details on fuzzy operators see papers [3,4,5,6] , books[2,9], and software [10].

2. Implementation

2.1 Fuzzy Set Creation

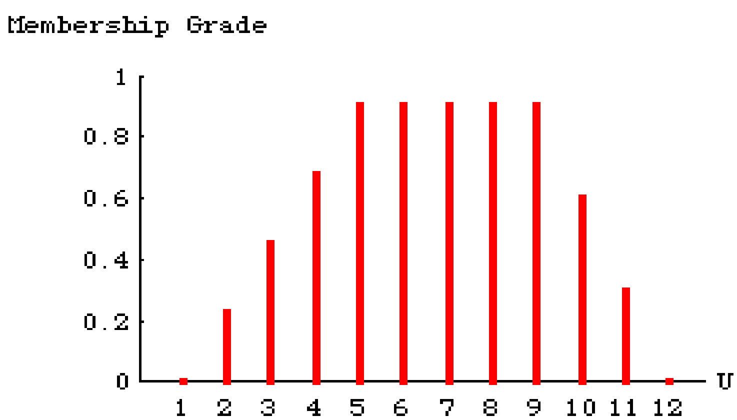
To define a fuzzy set A and set it equal to a twelve element fuzzy set the following would be entered into *Mathematica*®:

```
A  =  FuzzySet[{{1,1},{2,1},{3,.9},{4,.6},{5,.4},{6,.3},
        {7,.1},{8,0},{9,0},{10,0},{11,0},{12,0}}].
```

A fuzzy set can also be automatically generated using functions defined in the package. An example of creating a fuzzy set in this manner follows:

```
B = FuzzyTrapezoid[1, 5, 9, 12, .9]
```

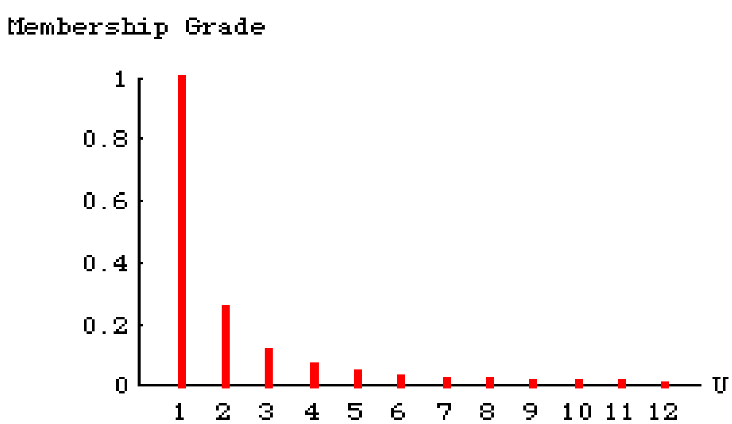
This creates fuzzy set **B**, a trapezoidal fuzzy set ranging over the universal space containing integers from 1 to 12 with peak plateau of 0.9 over the elements from 5 to 9. Graphically, fuzzy set **B** is:



Often it is desirable to create a fuzzy set from a mathematical function. To create such a fuzzy set a function must first be defined, say **F[x] = 1 / x²**. Using this function a fuzzy set can be created as follows:

```
C = CreateFuzzySet[F]
```

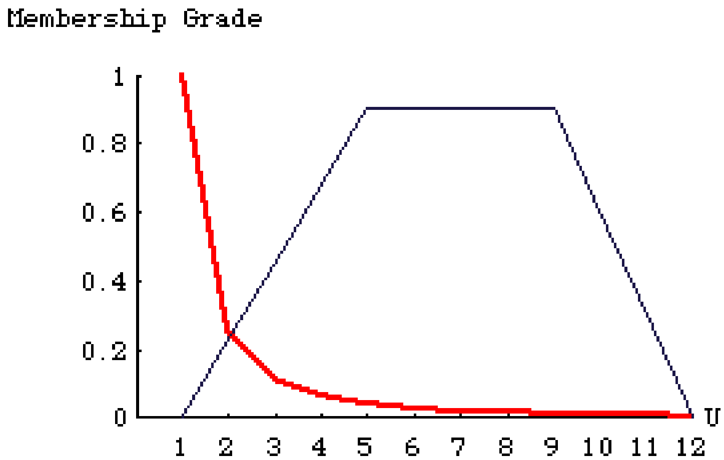
The graph of **C** is:



Fuzzy sets **A** and **B** can also be plotted in continuous form using the function call:

`FuzzyPlot[B,C,PlotJoined->True].`

The Graph is below:



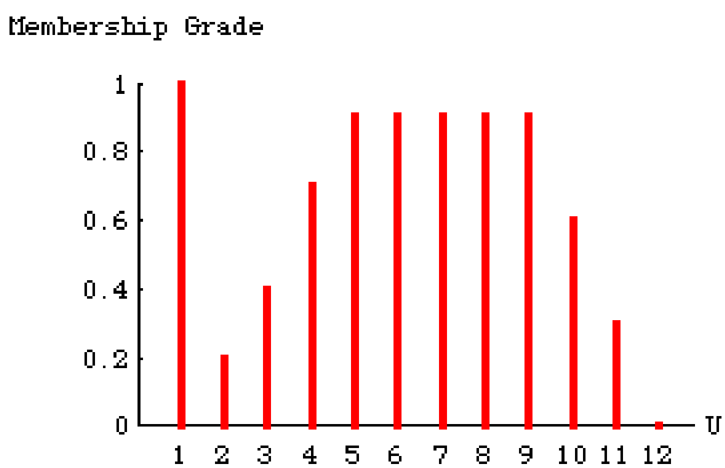
2.2 Fuzzy Set Operations

Once defined, fuzzy sets can be used with the various operator and manipulation functions included in the package. Fuzzy sets can also be displayed in digital or continuous form using the package's graphical capabilities. An example of performing the union operation on fuzzy sets **B** and **C** (defined above) follows.

To perform the union operation and display the resulting fuzzy set enter:

```
Result = Union[B,C]
FuzzyPlot[Result].
```

Then, the outcome would appear as:



2.3 Fuzzy Relation Definition

A fuzzy relation is defined in the package as a set of ordered triplets. An example of defining a fuzzy relation **R** (where only a part of the full definition of **R** is shown):

```
R = FuzzyRelation{...,{2,15,.5},{2,16,.5},{3,1,0},
{3,2,.25},{3,3,.5},{3,4,.75},{3,5,1},{3,6,.75},...}.
```

2.4 Fuzzy Relation Functions

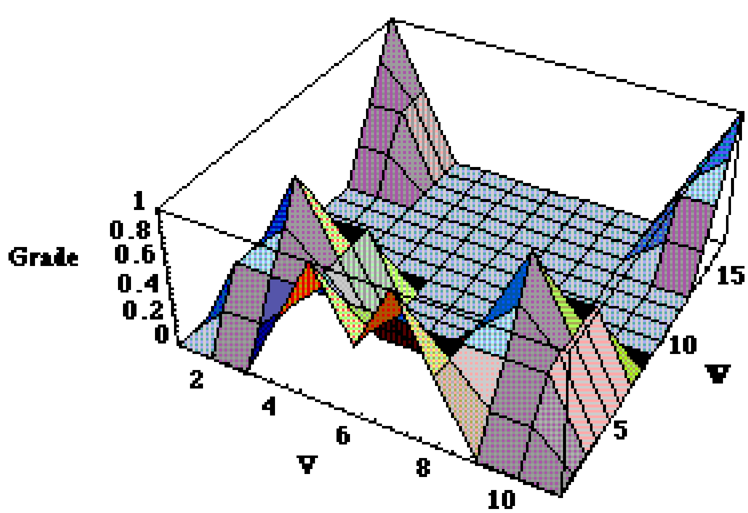
Within *Mathematica®* fuzzy relations can be combined using various operators or graphically displayed in a fashion similar to fuzzy sets. To display the fuzzy relation **R** in matrix form enter the following.

```
ToMembershipMatrix[R]
```

The resulting matrix is shown below:

0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1.0
0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0	0.25	0.5	0	0.5
0	0.25	0.5	0.75	1.0	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0	0	0	0	0
1.0	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0.5	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0
1.0	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0	0	0	0	0
0	0.25	0.5	0.75	1.0	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0
0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0	0.25	0.5	0.5	0.5
0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1.0

Fuzzy relations can also be displayed in digital form using the function **FuzzyPlot3D[R]**. This plot is similar to the digital fuzzy set plot except it is extended into three dimensions. The fuzzy set **R** can also be displayed as a surface using the command **FuzzySurfacePlot[R]**. The surface graph is below.



2.5 Complete Function Listing

Fuzzy set functions

Create a trapezoidal fuzzy set
Create a fuzzy set from a function
Create a Gaussian fuzzy set
Create a collection of fuzzy sets
Support set
Height
Alpha level set
Strong alpha level set
Cardinality
Relative cardinality
Equilibrium set
Concentration
Dilation
Contrast intensification
Max Union
Drastic sum
Bounded sum
Einstein sum
Algebraic sum
Hamacher sum
Min Intersection
Drastic product
Bounded product
Einstein product
Algebraic product
Hamacher product
Complement
Difference
Absolute difference
Symmetric difference
Arithmetic mean
Geometric mean
Harmonic mean
Plot digital fuzzy set
Plot continuous fuzzy set
Sugeno complement
Yager complement
Hamacher union
Frank union
Yager union
Dubois and Prade union
Dombi union
Hamacher intersection
Frank intersection
Yager intersection
Dubois and Prade intersection
Dombi intersection

Fuzzy relation functions

Create a fuzzy relation from a function
First projection
Second projection
Union
Intersection
Product
Algebraic sum
Complement
Disjunctive sum
Max-star composition
Max-min composition
Max-product composition
Display the first projection
Display the second projection
Display relation as a surface
Display digital relation
Display in matrix form

Defuzzification strategies

The Mean of the Maximum - MOM
The Center of the Area - COA

Fuzzy modeling

Fuzzy rules
Implication functions
Compositional rules of inference
Defuzzifications

Fuzzy clustering

Fuzzy control

3. Conclusion

This project's initial goal was to implement a software package to aid in the study of fuzzy logic. This goal was achieved when the package was used as a study aid for Dr. Stachowicz's ECE 5130 "Fuzzy Sets Theory and Its Applications" class at the University of Minnesota, Duluth, started in Fall 1992. However, the package has also shown that it can be used for many applications above and beyond the original intent. The ease with which digital fuzzy sets and relations can be entered and manipulated makes this package an invaluable tool for anyone interested in studying or applying the techniques of fuzzy logic. By requiring students to use the common functions from Fuzzy Logic Package, and encouraging them to take advantage of the various features of the *Mathematica*®, students gained an excellent understanding of the fuzzy sets theory and its applications in the mathematical modeling and control.

4. References

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