Mathematical competencies of engineering students

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Abstract — *In this paper we focus on the problem of mathematical competencies of engineering students. Results of some world examinations are cited. We present also some of our statistics collected in 2009 at the Silesian University of Technology where the correlation between final mark of mathematics, score of maturity exam and diagnostic test were examined.*

Index Terms —*Mathematical competence, education, ANOVA, correlation, multiple regression.*

INTRODUCTION

'Society needs a well educated population, to actively contribute to the shaping of society, and a broadly qualified work force, all of whom are able to activate mathematical knowledge, insights, and skills in a variety of situations and contexts. Yet, to an increasing extent young people opt away from educational programmes with a strong component of mathematics. At the individual level this is reflected in the so-called 'relevance paradox': Even though mathematical knowledge is highly relevant in and to society, many, if not most, people have increasing difficulty at seeing that mathematics is relevant to them, as individuals' [1].

This is also true in Poland and in many other countries. The 'maths problem' affects all levels of education, but we will concentrate on the tertiary education. The very serious problem that touches all technical universities in Poland is a very, very poor background in mathematics among students beginning the engineering education.

One could observe a continuous decline of students' preparation in mathematics during the past 25 years. There are two main reasons of this problem. 25 years ago the obligatory exam from mathematics was removed from maturity exam that finishes secondary education. This unfortunate decision has resulted in deterioration of the level of secondary education in mathematics. Consequently the majority of students entering technical universities are not prepared to study mathematics. For average citizen the above mentioned decision concerning maturity exam was a clear information given by authorities: 'mathematics is not so important' and also resulted in some mental change (which is much more serious), concerning the importance of mathematics. It is quite common opinion that mathematics is necessary only for a narrow group of people. This attitude is quite frequent even among the engineering students. They have their computers, programs and often do not feel the necessity of studying mathematics. Often they discover that mathematics is really necessary only when they reach higher level of education.

The second important reason of poor background in mathematics is the increase of the enrolment rate from 13% at the beginning of the nineties to more than 50% today. Unfortunately the increase of the enrolment rate was accompanied by deterioration of students' preparation to study mathematics. The transitional period in Polish economy after the fall of communism resulted also in growing unemployment especially among young adults with insufficient education It had also strengthened the incentive to continue education. Increasing the universities' quotas, foundation of many private universities have made entry into the universities much easier than in the past. Very often the motivation to choose technical studies is not the result of interest or abilities but difficulty to entry to another kind of studies (medicine, psychology, law, management, etc.). One can say that our students' mathematical competence is rather poor. But what is 'mathematical competence'?

MATHEMATICAL COMPETENCIES

According to Mogens Niss 'Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of intra-and extra-mathematical contexts and situations in which mathematics plays or could play a role' [1].
He introduced eight competencies. The first four are to do with the ability to ask and answer questions in and with mathematics, the rest are to do with the ability to deal with and manage mathematical language and tools. These competencies are as follows:

- thinking mathematically;
- posing and solving mathematical problems;
- Modelling mathematically (i.e. analysing and building models);
- reasoning mathematically;

- representing mathematical entities;
- handling mathematical symbols and formalisms;
- communicating in, with, and about mathematics;
- making use of aids and tools (including information technology) [1].

In order to improve educational system it is important to assess the performance of school students. This difficult task is taken up by PISA (the Programme for International Student Assessment). The programme started in 1997 with assessing the reading literacy (PISA 2000) of 15-year-old school students. The second assessment in 2003 was concentrated on mathematics and included also Polish students.

The PISA investigators examining competencies assess students attributing them one of six proficiency levels (the sixth is the highest). Being assigned to level 5 or level 6 is absolutely necessary to succeed studying engineering. In 2006 only 10.5% of 15-years-old Polish students were assigned to these two upper levels. These students will entry universities this year. Their one year older colleagues are finishing the first year of studies now.

From PISA 2006 documents results that Polish pupils have weaknesses such as: reasoning mathematically (analysis and deduction); argumentation; modelling mathematically; planning the solution of complex problem and strength such as: application of known algorithms; usage of graphs, diagrams, tables; geometric imagination [2].

Our didactic experience as university teachers with a long practice, confirms this diagnosis. It seems that 3 years of secondary education have not changed the situation. Still Polish students are better in routine tasks and have problems with coping with new ones that require creativity and reasoning. These weaknesses of Polish students result in passiveness, a tendency to learn by heart algorithms of solutions without understanding them, an unwillingness to individual work (especially homework).

A part of an adjustment of Polish Tertiary Education System to Bologna Process was a division of engineering studies into 2 cycles. This change has resulted in reduction of number of hours assigned for teaching mathematics. The number of hours of Calculus and Linear Algebra with Analytic Geometry was drastically reduced without changing study curricula that had been adjusted to at least 180 hours and to more extensive than present-day secondary education curricula.

Table 1 shows the total number of hours assigned for the lectures and classes from Calculus and Algebra on the courses of Computer Science in main Polish technical universities.

	The name of university	Total number of hours
1.	AGH University of Science and Technology	270
2.	Warsaw University of Technology	165
3.	Lublin University of Technology	150
4.	The Opole University of Technology	150
5.	Wrocław University of Technology:	
	- Department of Fundamental Studies	240 or 300 (according to student's choice)
	- Faculty of Computer Science and Management	120
6.	Technical University of Łódź	120
7.	Bialystok University of Technology	120
8.	Częstochowa University of Technology	120
9.	Gdansk University of Technology	120
10.	Koszalin University of Technology	105
11.	Silesian University of Technology	90
12.	Cracow University of Technology	90
13.	Poznan University of Technology	90
14.	Rzeszow University of Technology	75
15.	Kielce University of Technology	60

TABLE 1
TOTAL NUMBER OF HOURS OF CALCULUS AND ALGEBRA ON COMPUTER SCIENCE COURSE

The situation is very serious and difficult. On the one hand we have students with poor and very often dramatically poor background, on the other hand unchanged curricula (and these curricula do not take into account the change of secondary education curricula) and too limited number of hours.

In such situation teachers have two possibilities: to 'forget' about the curriculum and try to teach as much as possible considering the level of students or try to complete the curriculum being aware of the fact that majority of students do not follow up. The deterioration of standard of mathematical education for engineering students in such situation is inevitable.

The universities try to resolve this difficult situation, but it is impossible without extra funds. Fortunately since Polish entry to UE, there has been a possibility to participate in some operational programmes such as Human Resources, Infrastructure and Environment or Innovative Economy. But still it is drop in the bucket. Eight or sixteen additional hours of mathematics is not enough to help students with really serious problems. Studying Table 1, one can notice that some of the universities (or rather faculties) stress on the basic subjects and dedicate to mathematics more hours. This attitude benefits because students reach higher level of proficiency in mathematics that normally translate to success in engineering.

The Wrocław University of Technology has found an interesting solution for the problem of diversified secondary school qualifications and indecisiveness in the matter of choice in the field of studies. Instead of entering a specific faculty, an indecisive student can enter the Department of Fundamental Studies. During the first year he studies basic subjects such as mathematics (240 or 300 hours of Calculus and Algebra), physics, computer science. There is a possibility to choose the proper advancement level of each of these courses. It is also recommended to choose some more subjects according to one's interests.. A special adviser helps students to plan their own curriculum for the first year. At the end of the second semester students should make decision about the faculty and they can continue studies on the second year of a chosen course.

MATHEMATICAL COMPETENCIES OF SILESIAN UNIVERSITY OF TECHNOLOGY STUDENTS

The analysis of results of diagnostic test

At the beginning of October 2009 all first-year students of Silesian University of Technology wrote a short test which examined their mathematical knowledge. The test, called the diagnosic test (DT), was the part of realised European project 'I etap wdrożenia kompleksowego Programu Rozwojowego Politechniki Śląskiej w Gliwicach'. Students were asked to solve ten problems, namely:

- solve a quadratic inequality;
- find the domain and zeros of rational function;
- simplify an expression involving fractions, powers and square roots;
- test the divisibbility of some polynomial by a binomial;
- calculate a logarithm;
- solve a rational equation;
- test the monotonicity of geometric sequence;
- find the domain of a function containg trigonometric functions;
- find the inverse function;
- calculate the length of sum of two vectors.

All tasks were very easy. Neverless, the results were not satisfying. They are presented in Figure 1.

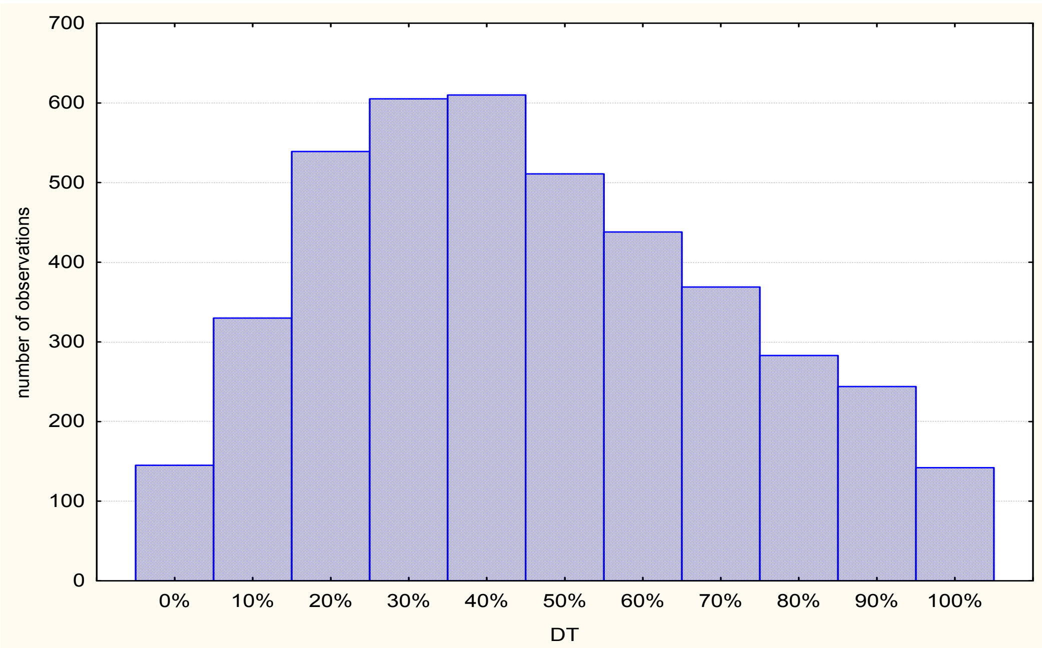

FIGURE 1
THE RESULTS OF DIAGNOSTIC TEST (DT)

For the whole sample of size 4216 the mean of DT was 45.80% and standard deviation 25.75%. We observed the differences between faculties. The detailed description is given in Table 2.

Faculty	N	Mean	Standard Deviation
Architecture (Arch)	173	75.38%	18.53%
Civil Engineering (CE)	346	70.23%	22.38%
Mathematics and Physics (M&P)	95	64.74%	23.06%
Automatic Control, Electronics and Computer Science (AE&CS)	534	60.36%	25.06%
Electrical Engineering (ElEng)	336	45.32%	23.26%
Mechanical Engineering (ME)	672	45.19%	22.97%
Organisation and Management (O&M)	293	43.69%	19.78%
Transport (T)	253	43.44%	21.26%
Energy and Environmental Engineering (E&EE)	689	33.32%	20.51%
Chemistry (Ch)	161	31.30%	19.82%
Mining and Geology (M&G)	458	30.96%	19.47%
Materials Engineering and Metallurgy (ME&M)	206	28.54%	19.43%

TABLE 2

RESULTS OF DT ON 12 FACULTIES OF SILESIAN UNIVERSITY OF TECHNOLOGY

In order to see if the mean result of DT is the same for each facullty, we employ the analysis of variance [3], [4].. For the experimental data the value of F test is F=156.62 with *p*-value close to zero (less than 0.01). Thus the null hypothesis, i.e. all means are equal, should be rejected. However, ANOVA is based on the assumption that variances of response variable are the same in each group. This assumption is not satisfied – in the Levene, C-Cochran and Bartlett tests obtained values of test statistics were too large. In such situation one can try to employ the Kruskal-Wallis nonparametric test. Then the value of the test statistic is H(11, 4216)=1182.11 with *p*-value less than 0.01 which means that there are differences between faculties. Next, procedure of multiple comparisons shows that there are four homogeneous groups: {ME&M, M&G, CH, E&EE}, {T, O&M, ME, ElEng}, {AE&CS, M&P }, and {M&P, CE, Arch}.

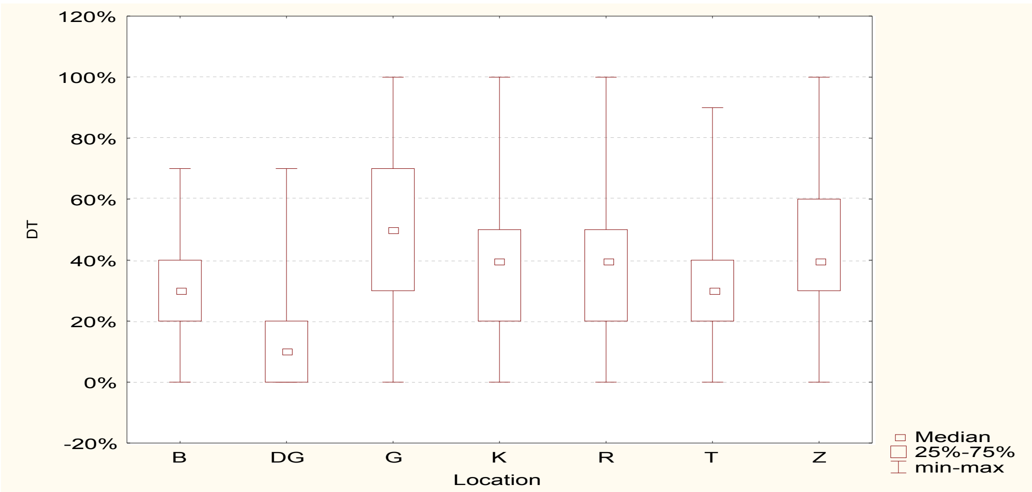

FIGURE 2

THE RESULTS OF DIAGNOSTIC TEST (DT) IN DIFFERENT LOCATIONS

Similarly, the Kruskal-Wallis test was used with another grouping variable – location (some faculties have students in a few towns). The best students (see Figure 2) are in Gliwice (G) and Zabrze (Z) – their means of DT are 48.18% and 46.03%, the worst means have students from Dąbrowa Górnicza (DG) and Bytom (B) – 17.65% and 30.43%, respectively. Table 3 shows *p*-values for multiple comparisons of pairs of locations (*p*-value less than α indicates the difference between groups; α is the significance level equal to 0.05).

	Bytom	Dąbrowa Górnicza	Gliwice	Katowice	Rybnik	Tychy
Bytom						
Dąbrowa Górnicza	1.000000					
Gliwice	**0.000042**	**0.000030**				
Katowice	0.709064	**0.025610**	**0.000000**			
Rybnik	1.000000	**0.039590**	**0.000000**	1.000000		
Tychy	1.000000	0.650770	**0.018596**	1.000000	1.000000	
Zabrze	**0.000920**	**0.000148**	1.000000	**0.001519**	**0.001339**	0.090535

TABLE 3

RESULTS OF MULTIPLE COMPARISONS (RESPONSE VARIABLE – DT, GROUPING VARIABLE – LOCATION)

Of course there are differences inside faculties. For example, at the Faculty of Automatic Control, Electronics and Computer Science the best results were obtained by students of Computer Science and Macro Course (the mean of DT equal to 76.91% and 67.87%), next were the students of Electronics and Telecommunications (53.36%) and Biomedical Engineering (41.61%). It follows from the post-hoc test that these three groups are separable at the significance level α=0.01.

Further examinations

For the best group (CS and MC) further examinations were conducted. We have compared results of DT with maturity exam (ME), the total score obtained by each student at classes during the first semester at Calculus course (Cl, in %), and final marks (FM). ME is the Polish obligatory exam for school leavers and it corresponds to A2 level. The final mark is the weighted mean of Cl and score in the exam. The pass mark, i.e. 3, is 40 out of 100.

For CS and MC students we have following characteristics:

- DT – mean 75.32%, standard deviation 18.35%,
- ME – mean 72.78%, standard deviation 15.52%,
- Cl – mean 44.49%, standard deviation 23.44%,
- FM – mean 3.14, standard deviation 0.97.

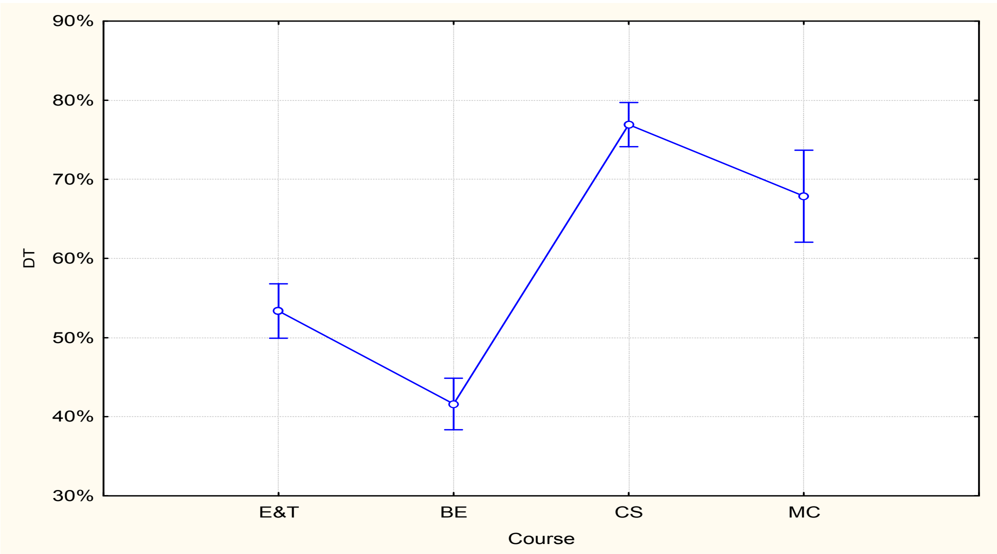

FIGURE 3
THE RESULTS OF DIAGNOSTIC TEST (DT) AT THE FACULTY OF AUTOMATIC CONTROL, ELECTRONICS AND COMPUTER SCIENCE

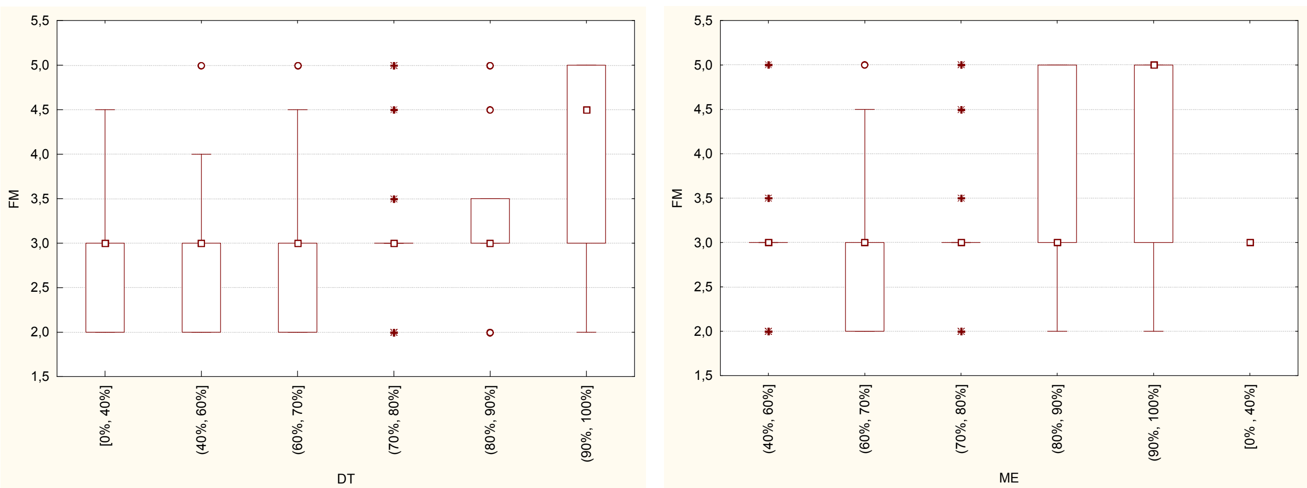

FIGURE 4
DEPENDENCIES BETWEEN FINAL MARK AND DIAGNOSTIC TEST OR MATURITY EXAM
(squares denote medians, boxes – first and third quartiles, circles – mild outliers, pluses – extreme outliers)

Although means of DT and ME are similar, the correlation coefficient is 0.51. The linear associations between FM and DT or ME are weaker – correlation coefficients are 0.38 and 0.42, respectively. Graphical interpretation is more suggestive. For example, in Figure 4 one can observe the median of FM equal to 3 in almost all posible intervals of DT and ME. Simultaneously there are 21 students with FM equal to 5 (the best mark) and 100% of DT as well as there are 59 students with FM=2 who obtained at least 60% of DT. The regression model of FM dependence on ME and DT is

$$FM = 2.148018\, ME + 0.803948\, DT + 1.106880 \pm 0.87953$$

(0.481061) (0.406309) (0.336599)

but its adjusted multiple coefficient of determination is $R_\alpha^2 = 0.1846$ (which means that only 18.46% of the variability in FM is explained by the equation above). In parentheses are given errors for the model coefficients.

The high linear association is observed between FM and Cl (0.89). We have the following linear relationship between FM and Cl:

$$FM = 3.660027\, Cl + 1.508214 \pm 0.44892$$

(0.120651) (0.060639)

with $R_\alpha^2 = 0.7848$. However the better model is that including all variables DT, Cl, ME:

$$FM = 3.903970\, Cl + 1.038435\, ME + 0.757689 DT \pm 0.55924$$

(0.225643) (0.276777) (0.239463)

This model explains 97.35% of variability in FM.

Conclusions

In general, students of Silesian University of Technology have rather poor backgrounds in mathematics. The independent diagnostic test (DT) conducted by university teachers confirms this because the mean of very easy and basic DT was 45.8%. Moreover, there were more students with lower results.

Although students of Computer Science and Macro Course of the Faculty of Automatic Control, Electronics and Computer Science entry the university with very good secondary school qualifications (high score of ME), there is no high correlation between their results of ME and results of DT at the beginning of an academic year.

Despite high score of ME, both students and teachers are disappointed by the weak final results of studying mathematics. During the semester there is realised ambitiuos curriculum but students have no sufficient time to be familiar with it. Some universities assign more hours weekly for mathematics classes and lectures (compare Table 1). It is important in the educational process because, according to PISA assessment, Polish students are not prepared to individual and creative work.

At the end let us notice that ME does not determine the final result of a mathematics course. There occur (not numerous) students with good ME score and poor FM, and vice versa.

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