Teaching algebra and calculus to engineering freshers via Socratic Dialogue and Eulerian sequencing

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Abstract — *Modern undergraduates join University courses in UK with poorer background than in the past. University tutors spend more and more time delivering stepping-stone classes. When doing so, most rely on traditional methods of delivery. However, such methods do not work when dealing with large groups of undergraduates who have limited background, limited memory, limited proficiency in explanatory reasoning, limited confidence, limited interest in the subject, limited study skills and on top of that, limited time to cover a large amount of materials, all aggravated by a limited contact with teachers. Yet, these disadvantages can be overcome when dealing with adult learners. Our specific aim is to present evidence that they can achieve relatively deep learning of mathematics – and remarkably quickly – through a teacher-guided Socratic dialogue, which aims to uncover learner difficulties and reinforce their basic understanding through Eulerian sequencing. We report common student misconceptions and also on progress in developing a Cognitive Tutor e-PACT (electronic Personal Algebra and Calculus Tutor) based on the above ideas.*

Index Terms --- Cognitive Tutor, Eulerian sequencing, Engineering mathematics, Socratic dialogue

INTRODUCTION

It is well documented that students are entering UK Universities with widely diverse educational backgrounds. Whilst traditional students have three or more A-levels, Widening Participation students may have equivalent qualifications from vocational or access courses or BTEC's [12]. Evidence has indicated that many of those experience difficulties with the transition to university and with progression generally [5]. On top of that, some traditional students meet the necessary minimum requirements but not necessarily have the required knowledge.

As the result, Universities spend considerable resources delivering summer schools, access, foundation and stepping-stone courses. Most teachers, who themselves had been taught in a traditional way, rely on traditional methods of delivery. However, such methods had been developed for the learners with good memory, good pattern recognition abilities, high confidence, and a considerable time to practice. These suppositions are particularly unrealistic when dealing with undergraduates who have limited background, limited memory, limited proficiency in explanatory reasoning, limited confidence, limited interest in the subject, limited study skills and on top of that, limited time to cover a large amount of material, all aggravated by a limited contact with teachers.

Yet, these disadvantages can be overcome when dealing with adult learners. The aim of this paper is to present evidence that they can achieve relatively deep learning of mathematics – and remarkably quickly – through SDES, a teacher-guided Socratic (but not sarcastic!) Dialogue, which aims to uncover learner difficulties and reinforce their basic understanding through Eulerian Sequencing. The latter is a name for verbalisation, a systematic approach to mathematics as a language that allows students to analyse (sequence) mathematical expressions and thus find the relevant solution algorithms (sequences of solution steps). This way teachers can learn to teach, and learners can learn to understand, why various steps are to be taken and not just what the steps are. We report common student misconceptions and also on progress in developing a Cognitive Tutor e-PACT (electronic Personal Algebra and Calculus Tutor) based on the above ideas.

THE SDES TEACHING METHODOLOGY

As mentioned in the Introduction, SDES is a teaching approach involving students in a Socratic Dialogue based on Eulerian sequencing. Originally, the concept of Socratic dialogue had been associated with literary works developed in Greece at the turn of the fourth century BC, preserved in the Plato dialogues, in which characters used the Socratic (question and answer) method to discuss moral and philosophical problems though at least one dialogue centred on mathematics: "Now, my young friend, tell me what is the object of mathematics? What does a mathematician study?" [17]. Towards the end of the last century, the method was extended by such educationalists as Collins [4], who introduced it into a general pedagogical discourse, and Hake [11], who revolutionised the teaching of undergraduate physics, allowing ordinary learners to master Newtonian mechanics.

The aim of Socratic dialogues is to achieve insights. Six facilitator measures for achieving the aim are suggested in [10]:

- holding back one own's opinion
- iliciting learners opinions
- working towards common understanding
- ensuring that the group focuses on the current question
- striving for consensus
- recognising and using fruitful questions and starting points.

In SDES methodology .Socratic dialogue is adapted to teaching mathematics to large classes of ordinary learners (it has been tried on classes up to 100 students). In this context the "dialogues" typically involve two speakers at any one time, with one (teacher) leading and structuring the discussion. The emphasis is on the fact that for learners to achieve insights into the subject the teachers have to achieve insight in their thought processes.

Standard **teacher reservations** expressed when faced with the idea of a teacher guided dialogue are:

1. Students feel intimidated when asked questions in class.
2. Socrates new that the answers to his questions existed and he knew the prior level achieved by his students.
3. A dialogue can work only in small groups.
4. Only some students will participate.
5. Students' attention might wander.
6. A dialogue should be student driven and not teacher driven.
7. It is difficult to take into account cultural differences.
8. If given an opportunity to opt out of the dialogue most will.
9. You can get off course.
10. Students learn best in groups rather than when guided by a teacher.
11. Students have different learning styles.

12 Teachers have to be trained to conduct Socratic dialogues.

These reservations can be dispelled using the following **seven golden rules of Socratic dialogue**:

1. **Students should know what to expect.**

There should be an introductory speech along the following lines:
"The lectures will be conducted in an interactive manner: it is very important for you to keep asking me questions and I would be asking you questions all the time. If I was in your place I would be terrified! Yet it is very important for you to learn to debate in front of many people. When you become engineers you will have to do this a lot. Also, it is important for both you and me to have feedback: I need to know what's in your heads, how fast or slow I should go and you need to know whether you are really achieving understanding."

2. **Students should be given a choice.**

At the end of the introductory speech, and from time to time, it is important to keep saying "I believe that you would all benefit from participating in a dialogue. However, if you feel that - for whatever reason - you do not want to, just tell me so before class."

3. **Hacking and spurious student-student dialogues should be actively discouraged.**

Sometimes students laugh at wrong answers given by others and sometimes they just talk between themselves. This should be actively discouraged by pointing out that this is an unprofessional behaviour and that it is very important to learn to work as a team. Students have to be explained than when they talk to each other for others this is sounds like a disturbing noise and this is never done at professional meetings.

4. **Student question should be actively encouraged.**

Every presentation of a small piece of theory and an example should finish with "Any questions?" If none are coming it is important to keep repeating something like "Remember, if you do not understand something, more than likely at least 30 % of other students do not understand this either. By asking questions you learn better and you help others to learn."

5. **Teacher questions should be kept very simple.**

At this level, the aim of Socratic dialogue is to ascertain that students understand basic concepts and logical steps, simultaneously uncovering and filling in gaps in their background knowledge

6. **There should be a large degree of engagement.**

SDES relies on three major techniques:

a) ask "Who thinks they know the answer to this question?", make mental note of the number of hands raised and pick one student to talk to, preferably the one who did not volunteer to answer. You will soon learn which student has problem with which topic, so you can monitor their progress.
b) offer an answer and ask "Who thinks the suggested answer is correct?", "Who thinks it is incorrect?", "Who does not know?" Encourage the "do not know" answer as professional. Comment on the number of correct answers (I often said "In the first year the majority is always wrong", which was always received with laughter).

c) suggest students shout different answers, right them on the white board. Then take a vote. Discuss what is wrong about wrong answers.

7. A teacher should maintain psychological control.

About 50 % of students have an immediate positive reaction to the interactive atmosphere of SDES classroom. Others need to get used to it. There is always a minority who feel threatened by it. It is extremely important for a teacher to

a) always have a calm and friendly demeanour,
b) NEVER allow bickering in class. If a student does not want to answer, asks "Why do you pick on me?" or voices any other complaint, immediately stop talking to him/her, make a pacifying joke and move on.
c) always wear pink to class! This has a strong pacifying effect of its own.

Rules 1, 2, 3, 5 and 7 deal with Reservation 1. Field notes and audio recordings of lectures conducted by independent researchers (Crisan and Lerman) who attended my classes confirm that students do not feel threatened by this strategy even if they do not always have an answer to the questions asked. They feel comfortable attempting to answer all the questions posed [7]. Rule 5 also deals with Reservation 2.

Following Rule 6 it is possible to overturn Reservations 3, 4. and 5 The video of the author's lecture shows her interacting with the class of 50. SDES was also practised in a class containing 100 students which presented no more problems than a class of 50. When students are talked to most of them stay naturally focused. They want to be able to answer and avoid "looking silly" in front of their peers.

Following Rule 4 a teacher can allow student initiative and deal with Reservation 6. The reservation comes from those who broadly follow Piaget's rather than Vygotsky's approach as applied to teaching adults [15]. Accepting the constructivist argument that every learner has to construct his or her own cognitive structure through their own effort and commitment, the question is how this can be achieved. The Piaget purists believe in "research based learning". This can work in other disciplines, when learners have a lot of time and also when teaching mature (extra-ordinary!) learners. However, it is unrealistic to expect that ordinary learners of mathematics who have limited time to master the subject can generate enough questions of quality that can lead to significant learning. enhancement. No student is going to ask to teach them to remove brackets or discuss what is meant by the word "constant"!

Reservations 7, 8 and 9 are understandable but the proposed approach was practiced in a Central London University which as such hosts an extremely diverse student population and various statistics reported in this paper show that most students respond to guided teaching in a positive manner. Very few students asked to be excused from the dialogue and this happened very rarely. It was easy to keep on course. Only a couple of times a year was there a need to say something like "We are running out of time and have to stop this discussion now. However, there will be opportunities to return to this point during tutorials and revision classes."

Reservation 10 is usually expressed by those who are unaware that ordinary learners benefit most when exercising individual responsibility for learning. When working in groups, the ideas of the more active students may dominate the group's conclusions. This is particularly evident in mathematics classrooms: unless actively engaged, slow learners fail to make the necessary connections [1], [20].

Similarly, Reservation 11 can be refuted by reference to recent pedagogical experiments that have shown that teaching by taking into account learning styles leads to no significant improvement - see recent articles on neuromyths, such as [18]. reservation 12 is valid and one of the aims of this paper is to promote understanding that University teachers need this specific type of training.

While Socratic dialogue assures continual student engagement and provides an immediate feedback to both teacher and student, the systematic approach to teaching mathematical abstractions to learners of unexceptional ability can be traced back to Euler. He believed that any student can be taught and enjoy basic mathematics, and for this reason the second building block of SDES is called "Eulerian sequencing". The sequencing is exercised in both making explicit the primary structures of mathematical expressions and in ordering solution steps. It is often supported by employing Decision Trees. It is in a perfect accord with recent pedagogical findings which suggest that, "The amount learned is proportional to the number of self-explanations that a student generates", the self-explanations being comments on a solution step "that contain… domain-relevant information over and above what was stated in the description of the step" [3]. To quote [6] "there are two general sources for self-explanations: The first is deduction from laws, rules, concepts and definitions acquired earlier, usually by simply instantiating a general principle, concept, or procedure with information relevant to the solution step. The second explanation is generalisation and extension of the step." Such construction of the content of the solution step yields new general knowledge that helps complete the students' otherwise incomplete understanding of the domain principles and concepts.

The practical outcome of SDES is that it allows students to solve not only the problems they repeatedly rehearsed before, but also problems that they have never seen. The learners acquire the ability to recognise familiar patterns in unfamiliar pictures, and thus are better equipped to become engineers rather than technicians.

Three standard **Teacher Reservations related** to the idea of Eulerian sequencing are as follows:

1. Ordinary learners cannot master abstract concepts
2. Students learn through constant practice
3. Different learners have different learning styles

Students raise seven standard **issues** with regard to Eulerian sequencing:

1. I was taught differently before
2. I have got used to a different language
3. I cannot use Decision Trees
4. I need more worked examples
5. I need more tests
6. I have time for nothing but training to exams. My colleagues at work cannot answer your questions, so there is no need for me to answer them.
7. I studied hard but there was no connection between what I studied and exam questions

Student Issues 1 – 5 are related to teacher Reservation 1. Student Issues 6, 7 are related to teacher Reservation 2.

To counter Teacher Reservations 1 and 2 let us first introduce **seven golden rules of Eulerian sequencing**:

1. **Do concept mapping**

At the beginning of each lecture, it is suggested that students are invited to do informal concept mapping by asking "What modules are covered in the Introductory Mathematics?", "What are the main concepts in Algebra?", "What are the main concepts in Calculus?" , "What types of variables do we study in Algebra?", "What type of functions do we study in Calculus?", "What operations on variables do we study in Algebra?" "What operations on functions do we study in Calculus?" and "Why do we study Algebra?" , "Why do we study Calculus?" The full concept map for algebra is presented in Table 1.

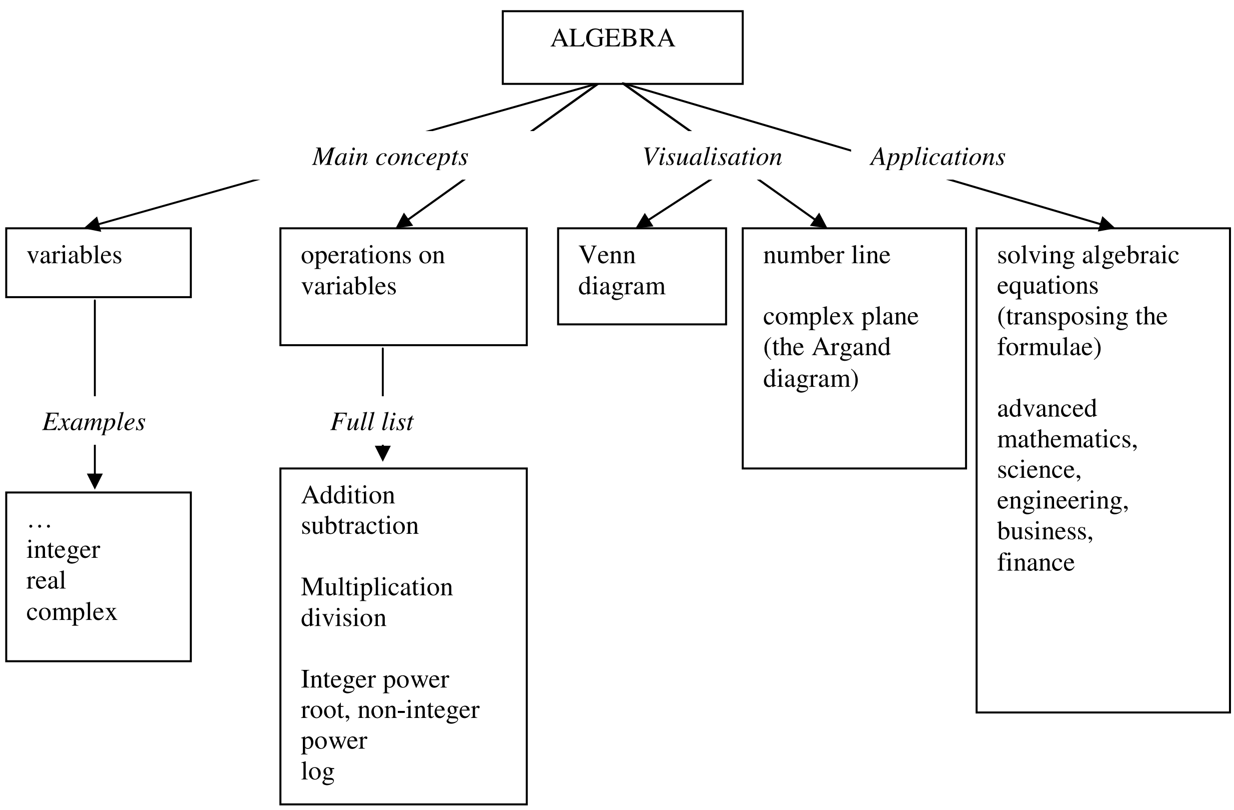

TABLE 1. CONCEPT MAPPING FOR ALGEBRA

2. **Ask questions aimed at reinforcing concepts**
"What does the word 'term' mean? ", "What does the word 'sum' mean? ","What does the word 'factor' mean? ", "What does the word 'product' mean? ", "What do we mean by a 'constant?", "What do we mean by a variable'?" "What does the word 'function' mean?"
3. **Ask questions aimed at reinforcing knowledge of methods and rules**
Typical examples are "What is the Smile rule", "What is a factoring rule?", "What is the order of operations?" "What is the product rule?"
4. **Ask probing questions**
Typical examples are "What methods of turning a product into a sum have we already learned?", "When factoring a sum, how do we find the first term, second term *etc.*?" "What are elementary operations on functions?"
5. **Ask questions aimed at sequencing expressions**
These questions are aimed at teaching students to itemise mathematical expressions the way a mathematician does. The examples are: "What is the first factor in this product?", "What is the second factor in this product?" "What is the differentiation variable in this problem?", "What is the function you are asked to differentiate?", "What is the constant factor in this term?"
6. **Ask questions aimed at teaching reasoning**
These are the questions of the Why? And What? type, such as "Why is this expression called a sum?", "What algebraic rule can be used to turn a sum into a product?"
7. **Ask questions aimed at sequencing a solution**
An ordinary learner is often baffled a to how start the solution process or how to proceed. The typical sequencing questions are "What is the next question to ask yourself?", "What step should you perform now?" This stage of teaching can be alleviated by employing a powerful scaffolding tool known as a Decision Tree.

Student Issues can dealt with respectively by the following explanations:

1. Human being have been designed to follow the first authority figure they encounter. Most prefer the first set of opinions and methods they have been exposed to. Education is about comparing different opinions and methods and then choosing the ones that suite you best
2. When learning a technical subject it is important to use a precise language. Maths school language is very imprecise.
3. Flow charts, sequential instructions, algorithmic approaches are routinely used by engineering students and practitioners. If a student finds them unintuitive this is the argument for and not against the necessity to train him/her to approach a problem in a systematic manner.
4. If a student understands what they are doing they need to do fewer exercises. If a student needs more exercise to boost their confidence there are many library and internet resources they can use to find extra exercises.
5. If a student understands what they are doing they need to do fewer
tests. If a student needs more tests to boost their confidence there are many internet resources available for this purpose.
6. Same as 4 and 5.
7. The purpose of SDES is to teach a student to recognise a familiar pattern in unfamiliar pictures. Education of engineers is not the same as training of technicians. You should not learn problems by heart but understand solution processes.

There is lot of practical evidence that the approach works: It comes from the author's practice who for 16 years taught *Introductory Engineering Mathematics* and coordinated teaching this subject to the first year engineering undergraduates of a UK New University. The unit provided students with the mathematical tools and methods needed in all other engineering programmes, namely, the basics of algebra of numbers, including complex numbers, and calculus. The unit had been taught over two semesters and delivered as a two-hour lecture every week. In addition, two-hour tutorials took place once a week. Lately, the minimum entry requirement at this University of widening participation was E in A-level mathematics (or equivalent). However, some students had a higher grade (up to B) and some mature students were accepted even without reaching E. Also, up to 26% of students could be dyslexic, dyspraxic or dyscalculic [16], most without being aware of their condition. The quality of teaching and learning has been partially evaluated by the author setting up exam questions that students have not seen before but could answer if they understood the material. It has been shown time and again [8], working with similar student intakes, that if both lectures and tutorials had been delivered via simple exposition promoting the traditional learning by rote the failure rate at the first attempt at exam would be 50% (could be 70%!) and if the methodology had been used to deliver both lectures and tutorials the failure rate would fall down to 30% (*ibid.*).

All SDES students approached by independent researchers [7] agreed that they needed time to 'get used' to SDES teaching, making statements like *"I was not used to explaining the mathematics"*. In the past they just "did it", without much verbalising or questioning as to 'why' and 'how'. One student with a secure mathematics background said that he found SDES teaching very different to how he was taught at the pre-university level. However, *"once you get used to the* *approach, it is OK; it is mainly the same thing but presented differently"*. All students agreed that this approach "*forced you to think, to really understand the mathematics"*. This evidence is supported further e.g. by the findings in [6] who conducted experiments on impact of dialogue and deep-level-reasoning questions in two studies with undergraduate students. They wrote:

"In Experiment 1, participants learned material by interacting with or by viewing one of four vicarious learning conditions: a noninteractive recorded version of the AutoTutor dialogues, a dialogue with a deep-level-reasoning question preceding each sentence, a dialogue with a deep-level-reasoning question preceding half of the sentences, or a monologue. Learners in the condition where a deep-level-reasoning question preceded each sentence significantly outperformed those in the other four conditions. Experiment 2 included the same interactive and non-interactive recorded condition, along with 2 vicarious learning conditions involving deep-level-reasoning questions. Both deep-level-reasoning-question conditions significantly outperformed the other conditions "

The above considerations counter Issues 1 – 2; with regard to Issue 2, also see recent articles on neuromyths, such as [18]. Both students with weak backgrounds and those who can perform most mathematical manipulations when they join the University find appeal in understanding what these manipulations mean. With regard to Issue 3, it is indeed true that ordinary learners find it difficult to exercise linear and algorithmic thinking and find Decision Trees counter-intuitive. However, this is an argument for, rather than against, teaching engineers to use such trees Issue 4 is countered by experience, which shows that a systematic approach to learning mathematics allows students to achieve learning faster rather than slower and this learning is deeper and thus, more effective than training to exams.

Last but not least, involving students in SDES allows teachers to discover common problems and misconceptions that are not always the same. They often appear or disappear following various changes school curricula. At one stage most new LSBU students had problems adding fractions. Most still cannot see the order of operations. None of them are comfortable with the concept of a function. Many of these problems are described in various documents published in [7].

e-PACT

While the teaching methodology described above puts a great emphasis on explanation of abstract mathematical concepts, acquisition of mathematics skills requires students to do a reasonable number of exercises and have their understanding of concepts and deep-level reasoning skills reinforced every time they make a mistake. This part of the educational process can be automated with a Cognitive Tutor System, a piece of software containing an artificial intelligence component to track students' work and tailor its feedback and hints, which captures the expertise of a specialist in a particular domain, therefore creating an artificial expert.

Two most prominent and relevant systems of this nature are AUTOTUTOR that is designed to conduct a Socratic dialogue with freshers studying Newtonian mechanics or IT [16] and a Carnegie Cognitive Tutoring System [2] that employs a similar approach to teaching algebra and geometry in schools. The author extended these ideas to more advanced algebra and calculus topics and produce a prototype of the first Cognitive Mathematics Tutoring System, an electronic Personal Algebra and Calculus Tutor (e-PACT). The current version of e-PACT is accessible via internet [7] and is aimed at teaching basic mathematics at the University level. e-PACT is meant to allow users to practice their skills at their own pace.

On the one hand, the e-PACT's cognitive model is to be simpler than any of those used in AutoTutor or CLS but on the other hand, unlike them, there is no reliance on database, for each topic, on each level of difficulty, it is to generate and discuss a large number of possible exercises (of prescribed types). This can run into hundreds. If the user inputs an incorrect solution he/she will to be told that the solution is incorrect and be given a chance to input the correct one; on the second failed attempt a student receives an explanation of what aspects of his/her answer are wrong and on the third one, all mistakes and the corresponding corrections will be listed in full. The student will then be offered an explanation and if the offer is accepted, e-PACT would take him/her through the solution step by step using SDSE. In particular, in response to each of his/her inputs the user would receive a specific message generated by e-PACT which either asks what question should the user ask themselves now, or asks a question, or else gives a specific explanation of the question or feedback to an answer.

Thus, e-PACT is designed to engage students in problem solving every step of the way, promote deep-level reasoning via self-explanation and a scaffolding tool not usually used in mathematics instructions, Decision Trees to allow students to navigate through solution algorithms and formulae. It contains Context Aware Parsers that recognise common errors and misconceptions and dynamic Decision Trees that sequence solution steps and guide the student through them with prompts and comments. e-PACT is to tailor its hints and responses to specific mistakes in student answers. Whatever student's turn, e-PACT is to present relevant and effective comments to build his/her mastery of the subject. By constructing

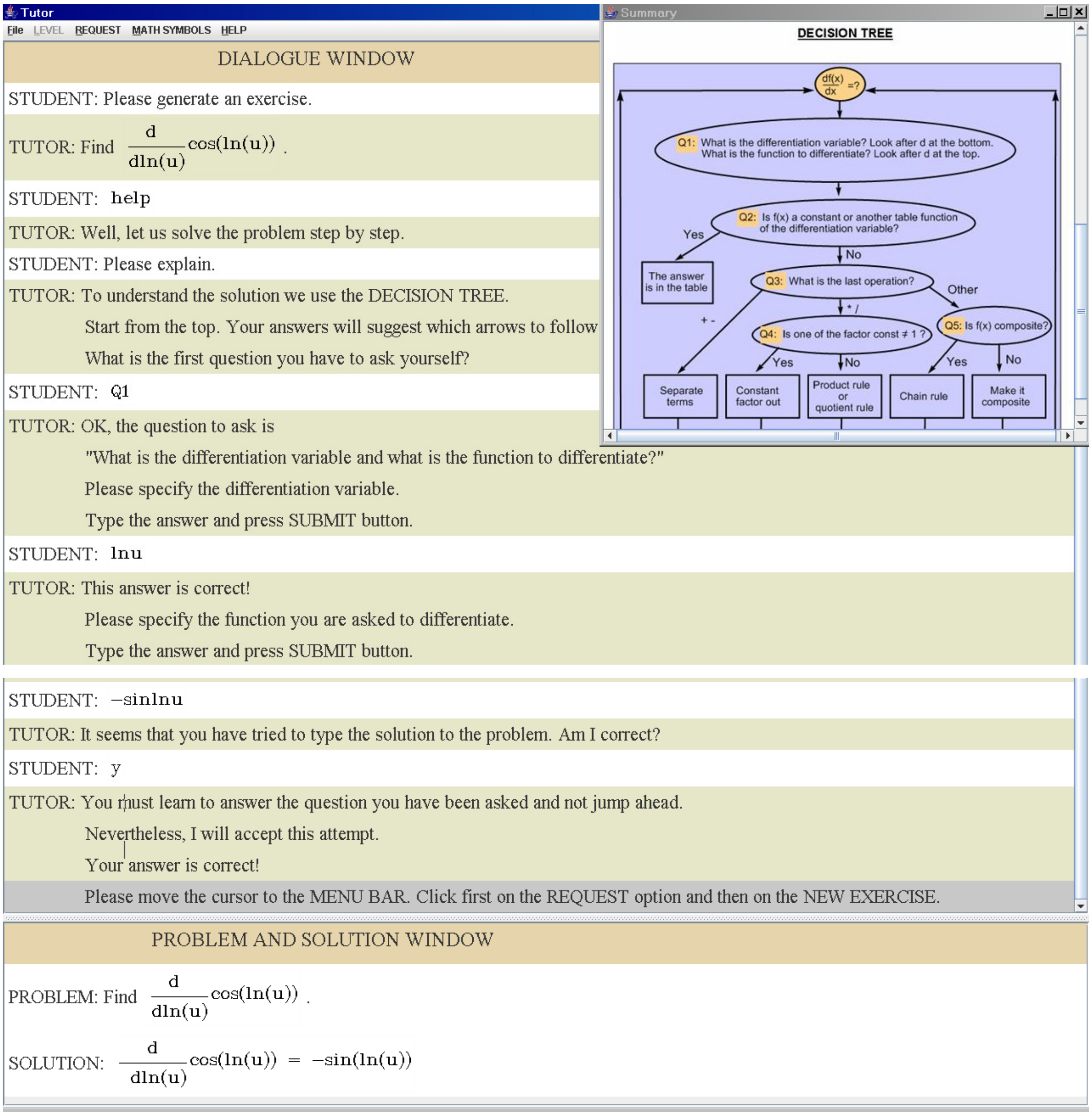

FIGURE 1. A SCREEN SHOT OF A 2007 VERSION OF E-PACT R WITH A SAMPLE DIALOGUE ON DIFFERENTIATION TOPIC, LEVEL 2.

human-like dialogues and using correct verbalisation of mathematical processes e-PACT is meant to emphasise importance of communication skills.

Whether commenting on a particular answer or engaging student in a Decision Tree based dialogue, it is a challenge to ascertain that an intelligent feedback is provided in all cases, and that that this is always done in an understandable and conversational manner. The problem is compounded by the fact that e-PACT is meant to interpret many different styles of mathematical input, allowing for many different conventions and a lot of sloppiness. This is a conscious choice, since "in tutorial systems, effective progress in teaching the problem-solving target is frequently hindered by expressive sloppiness and low-level errors made by the student, especially in conventionalized expressions such as formulas." [13]. Thus, the current e-PACT prototype is tolerant to a number of spaces used by the user, it can interpret a function whether the argument is bracketed or not (*sin(x), sinx* and *sin x* would be treated the same); if an expression is bracketed more than once, it just generates a gentle warning that the input contains extra brackets; and if a bracket is missing here or there, this is also handled through warnings rather than error messages. If an error is of the type expected of a dyslexic student, say the input is ex rather than e^x the prototype sends a detailed message on the corresponding mathematical convention, drawing the student's attention to the fact that in mathematics the order and position of symbols is often imbued with a particular meaning. The e-PACT prototype already "knows" enough algebra to be able to comment on such input as p^{-1} whether it is given in that form or as $\frac{1}{p}$ and in its messages will use the language appropriate to the form chosen by student. While this tolerance provides for better usability it makes interpreting the student performance and arriving at instructional decisions based on this interpretation a challenging task.

A sample screen shot is presented in Figure 1.

CONCLUSIONS

Promoting the teacher guided Socratic dialogue based on Eulerian sequencing, the SDES methodology is adapted to modern times to teach mathematics to large groups of engineering undergraduates, mostly with very poor mathematical background, about 26% of whom may be dyslexic, dyspraxic or suffer from dyscalculia.

The methodology ensures that students develop correct study skills, are taught rather than trained, master algorithmic and iterative approaches to problem solving and last but not least, learn the art of technical debate. The author is developing a cognitive tutor e-PACT that incorporates aspects of the above methodology.

Last but not least, the methodology allows teachers to discover common problems and misconceptions that are not necessarily always the same. Thus, not only does SDES allow learners to enhance learning, it also allows teachers to enhance teaching.

REFERENCES

[1] Anderson, J, R, Reder, L, M and Simon, H, A "Applications and Misapplications of Cognitive Psychology to Mathematics Education"., *Texas Educational Review,* 2000 Summer.

[2] Carnegie Learning http://www.carnegielearning.com Pittsburgh, PA | 1-888.851-7094, U.S.A., 2004.

[3] Chi, M, T, H and VanLehn, K, A, "The Content of Physics Self-Explanations", *Journal of Learning Sciences*, 1 (1), 1991, 69-105.

[4] Collins, C "Teaching reasoning skills". In Chipman, S, Segal, J & Glaser, R (Eds.). *Thinking and Learning Skills. Vol 2.* 1985, pp. 579-586.

[5] Cox, W, "On the expectations of the mathematical knowledge of first-year undergraduates", *International Journal of Mathematical Education in Science andTechnology*, Vol 32, No 6, 2001, pp 847-861.

[6] Craig, S, D, Sullins, J, Witherspoon, A and Gholson, B, "The Deep-Level Reasoning-Question Effect: The Role of Dialogue and Deep-Level-Reasoning Questions During Vicarious Learning". *Cognition and Instruction*, Vol 4, No 4 , 2006, pp. 565-591.

[7] Fradkin, L http://www.e-PACT.org, 2010.

[8] Fradkin, L. *Unit Monitoring Reports: Maths and Circuits*, ECCE, SBU, 1996 – 2004; *Unit Monitoring Reports: Introductory Engineering Mathematics*, FESBE, LSBU, U.K. 2005, 2006, 2007; *Unit Monitoring Reports: Introductory Mathematics,* FESBE, LSBU, U.K. , 2008, 2009.

[9] Fradkin, L, Zernov, V, Crisan, C and Lerman, S "First Year Engineering Mathematics: The London South Bank University experience". *MSOR Connections*, 2010 Spring .

[10] Hackman, G, "Six pedagogical Measures and Socratic facilitation". In Enquiring minds: Socratic dialogue in education (Eds. Saran, R and Neisser, B), 2004.

[11] Hake, R, R "Interactive-engagement vs. traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses", *American Journal of Physics*. Vol 66, 1998, pp. 64-74.

[12] HEFCE, "Strategic Plan 2006-11", 2007
Available online: http://www.hefce.ac.uk/pubs/hefce/2007/07_09/07_09.pdf [Accessed 18th July, 2007]

[13] Horacek, H and Wolska, M "Handling Errors in Mathematical Formulas", *Lecture Notes in Computer Science*, Vol. Vol. 4053, 2006, pp. 339-348,

[14] Parson, S,. Engineering Mathematics and the Dyslexic Student, presentation on 16 March 2004, 2004.

[15] Pass, S, "Parallel paths to constructivism: Jean Piaget and Lev Vygotsky", 2004.

[16] Person, N, K, Graesser, A, C, Kreuz, R, J, Pomeroy, R, J and the Tutoring Research Group, "Simulating human tutor dialog moves in AutoTutor". *International Journal of Artificial Intelligence in Education*, Vol 12, 2001, pp. 23-29.

[17] Renyi, A., *Dialogues on Mathematics*, 1967..

[18] Shepard, C. (2007)
http://66.102.9.132/search?q=cache:vjFOxGaYnh8J:cliveshepherd.blogspot.com/2007/06/neuromyths.html+neuromyth+learning+styles&cd=2&hl=en&ct=clnk&gl=uk

[19] Smith, A. "Making Mathematics Count. Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education", 2004.

[20] Tuckman, B, W,, 1982,
http://eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/30/18/99.pdf;