Valuable mathematical tools in engineering education

M. Leindl, E. R. Oberaigner
Institute of Mechanics, University of Leoben, Austria
ICEE-2010, Gliwice, Poland

Overview

Leoben

University of Leoben

Motivation

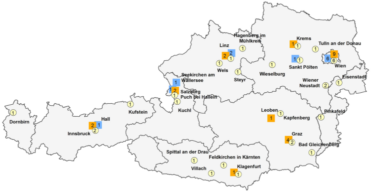
Problem description

Analytical solution

Solution with MAPLE^{TM}

Conclusions

City of Leoben, http://www.leoben.at/



Leoben, second largest town of styria, $\approx$ 25.000 residents; a center of heavy industry, Erzberg, voestalpine Stahl Donawitz (LD-process); a center of modern technologies; a city of conventions, culture and tourism.

M. Leindl, E. R. Oberaigner
Institute of Mechanics, University of Leoben, Austria

University of Leoben, http://www.unileoben.ac.at/

Short history

1840 Foundation as „Steiermärkisch-Ständische Montanlehranstalt“in Vordernberg
1849 Relocation to Leoben
1904 Renaming into „Montanistische Hochschule“
1975 Renaming into „Montanuniversität Leoben“

Studies

- Applied Geosciences
- Industrial Environmental Protection, Waste Disposal Technology and Recycling
- Industrial Logistics
- Master Programme Industrial Energy Technology
- Materials Science
- Natural Resources Engineering
- Mining and Metallurgical Machinery
- Metallurgy
- Petroleum Engineering
- Polymer Engineering and Science

Currently ≈ 2500 students, 44 institutes/departments, ≈ 500 first-year students in fall 2009

M. Leindl, E. R. Oberaigner
Institute of Mechanics, University of Leoben, Austria

Motivation

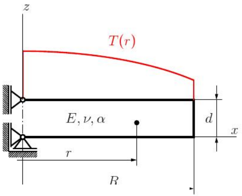
Why is the knowledge of tensor calculus and a CAS (computer algebra system) important?

- Advantage of formulating equations in continuum mechanics with the tensor formalism will be shown
- Equations in tensor form can be written in arbitrary coordinates suited for the considered problem
- Extensive calculations which are arising are done with a CAS - in our case: MAPLE™
- Approach allows to concentrate on concepts and not on time consuming calculations

Problem description 1

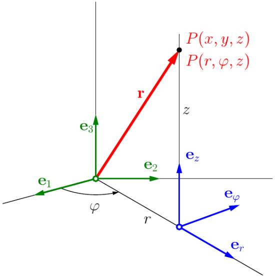
- Starting from basic principles the Lamé-Navier equations are calculated
- Stress-, strain- and displacement-fields in a circular disc or cylinder loaded by different radial temperature distributions are investigated
- Dependent on the ratio of thickness to radius a plane stress or plane strain state is present
- Solutions for the plane stress case are shown, obtaining the plane strain solutions is straightforward

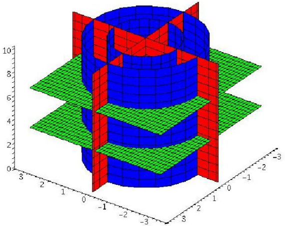
Problem description 2



$\frac{d}{R} \ll 1 \ldots$ plane stress
$\frac{d}{R} \gg 1 \ldots$ plane strain
$\frac{d}{R} \approx 1 \ldots$ 3-D-state

$E = 2.1 \cdot 10^5\ \mathrm{N/mm^2}$
$\nu = 0.3$
$\alpha = 12 \cdot 10^{-6}\ \mathrm{1/K}$
$R = 11.25\,\mathrm{mm}$
$d = 1\,\mathrm{mm}$
stress and strain free state at $T^\star = 273.15\,\mathrm{K}$





M. Leindl, E. R. Oberaigner
Institute of Mechanics, University of Leoben, Austria

Analytical solution 1

Complete set of field equations in the theory of linear elasticity:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_i|_j + u_j|_i\right), \quad \sigma^{ij}|_j + f^i = \rho \ddot{u}^i, \quad \sigma^{ij} = \frac{E}{1+\nu}\left(\varepsilon^{ij} + \frac{\nu}{1-2\nu}\varepsilon^m_m g^{ij} - \frac{1+\nu}{1-2\nu}\alpha_T T g^{ij}\right),$$

$$\varepsilon_{ij}|_{kl}\; e^{ikm} e^{jln} = 0 \; .$$

Lamé-Navier equations:

$$\mu u^i|^j_j + (\lambda + \mu) u^j|^i_j - (3\lambda + 2\mu)\alpha_T T|^i = 0 \; .$$

Due to symmetry conditions the Lamé-Navier equations are reduced to an ordinary differential equation:

$$\mathrm{d_r}\left(\frac{1}{r}\mathrm{d_r}(r u_r)\right) = \alpha(1+\nu)\mathrm{d_r} T.$$

Analytical solution 2

Stresses and displacement:

$$\sigma_{rr}(r) = \alpha E \left(\frac{1}{R^2} \int_0^R \mathrm{d}r \; Tr - \frac{1}{r^2} \int_0^r \mathrm{d}r' \; Tr' \right) , \sigma_{\varphi\varphi}(r) = \alpha E \left(\frac{1}{R^2} \int_0^R \mathrm{d}r \; Tr + \frac{1}{r^2} \int_0^r \mathrm{d}r' \; Tr' - T \right),$$

$$u_r(r) = \frac{\alpha}{r} \left[(1-\nu) \left(\frac{r}{R} \right)^2 \int_0^R \mathrm{d}r \; Tr + (1+\nu) \int_0^r \mathrm{d}r' \; Tr' \right] .$$

Stresses at $r = 0$ by application of *l'Hôspital's* rule to the limits:

$$\lim_{r \to 0} \frac{1}{r} \int_0^r \mathrm{d}r' \; Tr' = 0 , \quad \lim_{r \to 0} \frac{1}{r^2} \int_0^r \mathrm{d}r' \; Tr' = \frac{1}{2} T(0) .$$

$$\sigma_{rr}(0) = \sigma_{\varphi\varphi}(0) = \alpha E \left(\frac{1}{R^2} \int_0^R \mathrm{d}r \; Tr - \frac{1}{2} T(0) \right) .$$

Solution with MAPLE^{TM}

- Tensor calculus (base vectors, metric coefficients, Christoffel symbols, differential operators, ...)
- Calculation of the Lamé-Navier equations
- Solution of the differential equations for the field variables
- Evaluation of integrals and limits

MAPLE^{TM}worksheets

Conclusions

- Basic equations of linear thermoelasticity were developed and formulated in tensor notation
- Lamé-Navier equations were specialised for cylindrical coordinates
- A problem of linear thermoelasticity was solved analytically; different boundary conditions were investigated
- CAS was intensively used which takes care of all the time consuming calculations
- CAS helps to quickly get an impression how equations and their solutions behave; the saved time can be used for discussions of the results or case studies

Questions?

Thank you for your attention!

M. Leindl, E. R. Oberaigner
Institute of Mechanics, University of Leoben, Austria