# The role of mathematics in teaching engineering sciences modules

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## INTRODUCTION

The analysis and design of engineering systems calls for mathematical models that encapsulate key parameters and applicable laws [1]. Engineers routinely study processes, build systems and interpret experimental measurements, which at times are critical tasks that require an understanding of important mathematical skills, in spite of the widespread use of commercially available mathematical software for engineering purposes. Although some physical laws may be qualitatively described in the teaching of engineering sciences - such as thermodynamics, fluid mechanics, circuits theory - a precise quantitative statement may only be expressed in the language of mathematics [2], [3]. Mathematics is therefore indispensable to the engineering community, but the depth of its study is bound to be limited. The best 'practical' approach to mathematics is to understand it as a language for describing physical and chemical laws. Hence, from this point of view understanding an engineering problem means the conversion of this problem into a physical and/or chemical problem, and its formulation in terms of mathematical equations [4], [5].

However, it is a mater of fact that many academics in engineering departments acknowledge a steady decline in the essential mathematical skills possessed by new student intakes on their degree programs over recent years. A growing number of students who come into engineering programs have to cope with a large amount of mathematical contents with serious shortcomings in several key areas, which entails a risk of being left behind and that can lead to withdrawal from the course or failure. Nevertheless, engineering modules could never be purged of mathematical content without seriously diluting engineering knowledge, making its treatment superficial and ultimately undermining core aspects of graduate engineering.

This paper presents a case study of the integration of mathematics in several engineering modules. The approach treats mathematics as a useful problem-solving tool rather than an end in itself, placing the emphasis on challenging engineering problems. After gaining a clear understanding of the underlying engineering concepts, students can then take advantage of mathematical modelling and analysis. A team of teachers with the Department of Electromechanical Engineering and the Department of Mathematics and Computation of the University of Burgos have worked together to develop a number of integrated modules on mathematics and engineering. The aim of these modules is to improve the ability of students to understand and apply the mathematical models used in engineering sciences. They have been used to teach a Mathematical Methods course in the fifth semester of the Electronic Engineering degree at the University of Burgos. Some examples related to Theory of Circuits are described. Finally, perception questionnaires were used to measure students' opinions of the experiment.

## CASE STUDY

Teachers from various departments participated in a project over the 2004/5 academic year, which was intended to help students form direct connections between the contents of mathematics modules and other subject modules that form part of the course leading to a degree in Mechanical Engineering and Electronic Engineering at the Escuela Politécnica Superior de Burgos. In the first place, they studied the interdisciplinary relations between subject modules from the same qualification. To do so, they compared the contents of various subject modules and set out the mathematical knowledge needed to be able to learn scientific topics in a satisfactory way that form part of the electro-mechanics subject module. They thought that, at times, the differences in the language or notation used to refer to the same concepts in certain subjects might represent a difficulty for students when trying to identify the essential relations in order to improve their learning. They also studied the advisability of setting problems framed in the language of electro-

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mechanics in the mathematics modules and, at the same time, to stress the mathematical solutions in the electro-mechanics subject modules.

The time to put these ideas into practice came in the second semester of the 2004/05 academic year, in the optional subject module "Mathematical Methods", part of the third year of the course leading to qualification as an electronic engineer.

The subject module appeared very suitable, as it is optional, which implies an interest in mathematics among the students that enrol, and because, the opinions of students who had attended more classes on the course and who had wider experience of other subject modules would be consulted, as it is part of the third year.

To that end, the teachers set the students problems which were at all times framed in the language of electro-mechanics; specifically, from the subject module entitled Circuits Theory, part of the second year Electronic Engineering course. They related the subject modules and sought a solution to the problems using mathematical methods in keeping with the subject module they were teaching.

This type of presentation implied an additional effort of coordination between the teachers involved, so as to have the results prepared beforehand, especially the mathematics teachers, so as to be able to solve at least the basic questions taken from the engineering subject modules. In any case, it could be observed that the students, in general, tended to resort to the tutorials to ask for guidance from the engineering professors who, at an earlier point in time, had taught the related engineering subject modules.

In view of the preliminary results of this first trial, it was decided to continue the study, as the initial response from students had been a positive one. The following decisions were taken:

1. Given that frequently it is not possible to set interrelated problems simultaneously in two parallel subject modules, nor to teach subject modules with the simultaneous presence of mathematics and engineering teachers, it was thought appropriate to provide one set of notes for the two subject modules, which is to say interdisciplinary notes, pursuing the final objective of enabling the students to identify the coincidental points with greater ease.
2. Problems would be posed in the same terms in subject modules with shared notes. For the subject modules on mathematics, by making a reference to the origin of the problem, though the mathematical problem was directly set out. (Ex: Obtain the variables ….. of the given equations, knowing that they correspond to the following case …..belonging to the study module …..), while the same problem would be expressed in standard terms in the parallel engineering subject module (Ex: Calculate the variables….given the following data … assuming that ……).

As a consequence of the aforementioned stages described above, a defined structure was established on integrated joint notes for different subject modules. At present, these notes exist for the Circuits Theory and Special Mathematics subject modules, from the same second year of Electronic Engineering. The first is taught throughout the year and the second only in the first semester. All of the envisaged notes were prepared and made available to students through Burgos University's virtual teaching platform, UBUCampus-e, over the academic years 2006/07 and 2007/08. Fig. 1 shows the screen display of this learning management system.

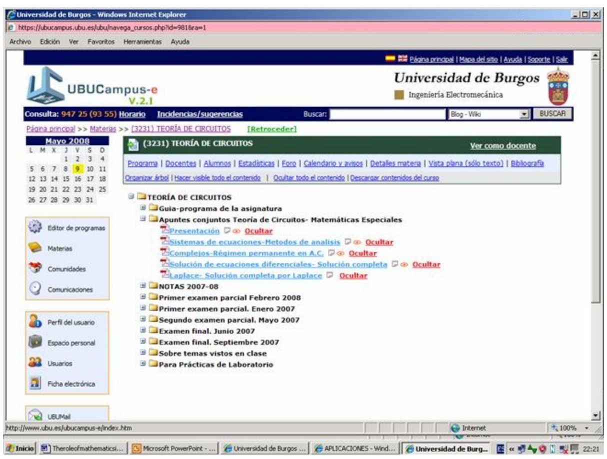


Fig. 1. Main screen of the Circuits Theory subject module at the UBUCampus-e.

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Joint notes were presented in both subjects in a folder entitled "*Apuntes conjuntos Teoría de Circuitos-Matemáticas especiales* [Joint notes Circuits Theory-Special Mathematics]" with the following content: Presentation; Section 1: Equations systems – Analytical Methods; Section 2: Complex numbers - AC stationary regime; Section 3: Resolution of differential equations –Complete solution; Section 4: The Laplace transformation - Complete solution for the Laplace transformation. These four sections establish the complete solutions between the two subject modules. Fig. 2 shows this interrelation.

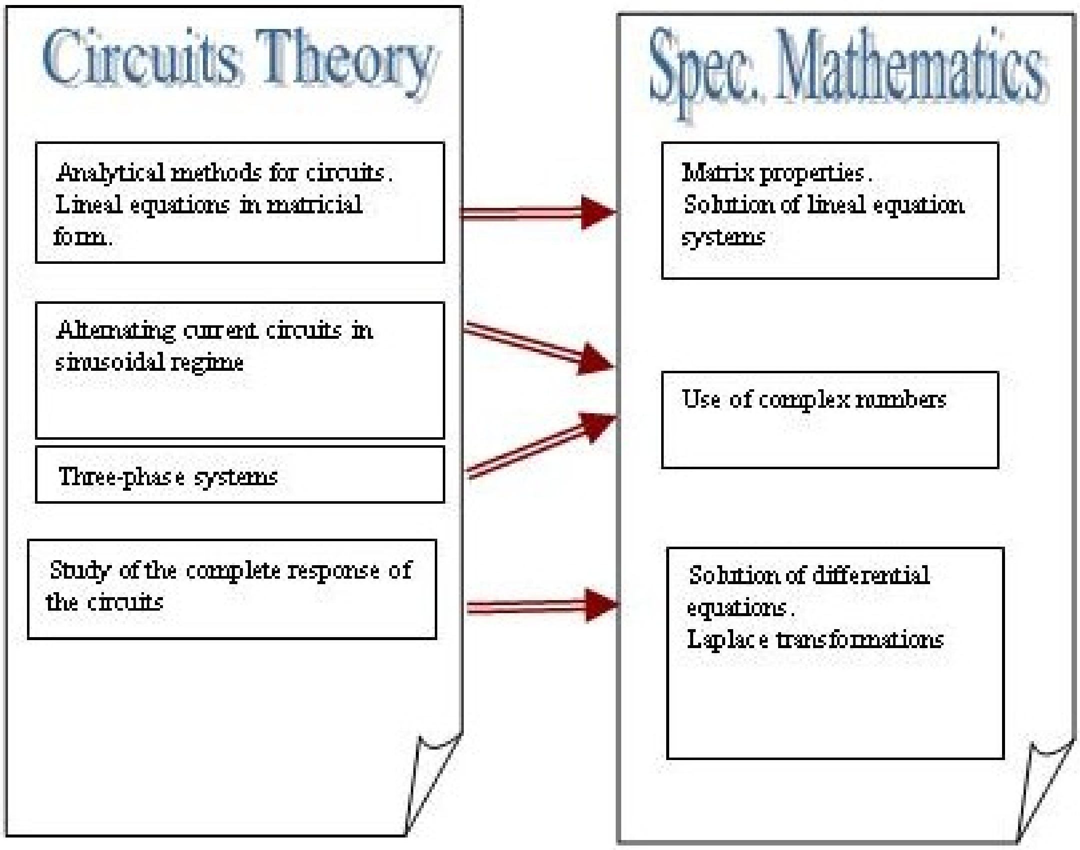


Fig. 2. Interrelations between subject modules, used for the preparation of joint notes.

The internal structure of each section is similar: The section heading is defined and the theoretical bases of both subject modules are set out and connected; the parallel forms of language are presented in tables and with common exercises. Table 1 is an example of a parallel presentation in table form.

Table 1. Joint notes on Circuits Theory and Mathematics. Example of first-order circuits.

| **Circuits Theory** | **Mathematics** |
|---|---|
| First-order circuits: Follow a first-degree differential equation. Have a single energy-storage element ( C o L) | First-degree differential equations |
| $\frac{dx(t)}{dt} + a.x(t) = f(t)$ | $\frac{dx(t)}{dt} + a.x(t) = f(t)$ |
| a = inverse of the circuit time constant | a = coefficient of x(t) |
| f(t) the dependent term of the circuit sources | f(t) the second term of the equation |
| Conditions at the exact time of an interrupter position change | Initial conditions |
| to-.= time immediately prior to 'to.' | to - $\varepsilon$ |
| to+.= time immediately after 'to.' | to + $\varepsilon$ |
| Complete answer $X(t) = X(t)_{\text{Natural response}} + X(t)_{\text{Steady-State or forced response}}$ | Calculation of the unknown factor x(t) $X(t) = X(t)_{\substack{\text{complementary sol.} \\ \text{or homogeneous eq. sol.}}} + X(t)_{\text{particular integral solution}}$ |
| Natural answer $X(t)_{\text{Natural response}} = A.e^{-at}$ | Solution to the homogeneous equation |

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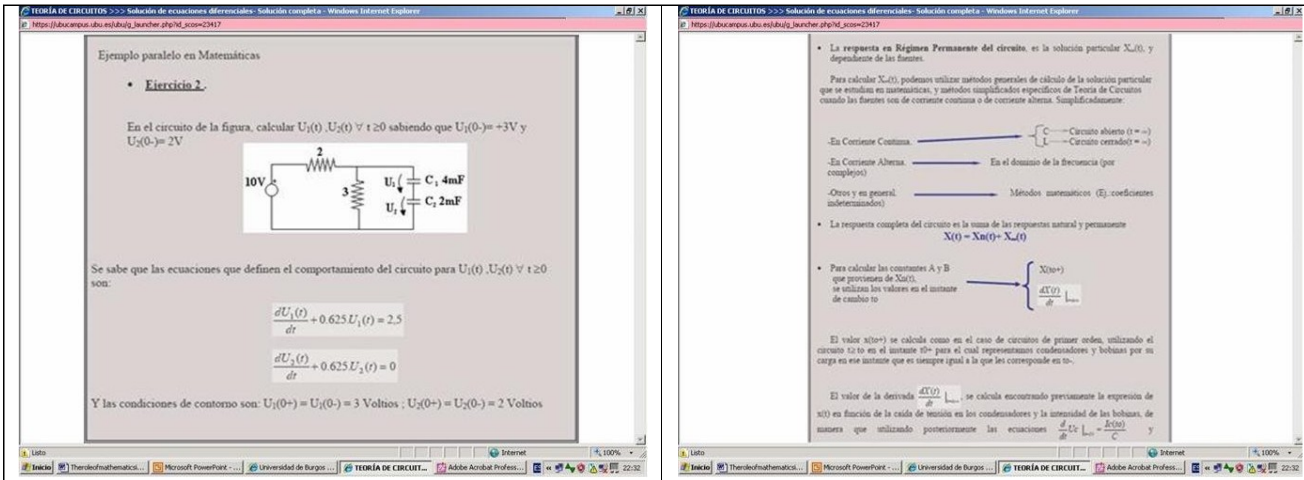
| | |
|---|---|
| | $X(t)_{\text{Complementary sol.(Ec.with f(t)=0)}} = A.e^{-at}$ |
| Permanent or forced answer<br>On the circuit using standard methods:<br>-If D.C. with capacitators in an open circuit and coils as short circuits<br>-If A.C. move to the frequency domain (calculated with complex numbers), perform calculation and return to the time domain<br>In other cases, only mathematical methods | Particular solution<br>Using standard mathematical methods:<br>-Indeterminate coefficients or<br>- $X(t)_{\text{ particular integral solution}} = e^{-at}\int e^{at} f(t)dt$ |
| Calculation of the A constant<br>X(0+) must be calculated taking into account the initial charge of the coils and capacitators | Calculation of the A constant<br>Circuit conditions will apply |

Fig. 3 shows examples of the exercises set for students following the Circuits Theory Subject Module on the UBUCampus-e platform, with the notation and methodology coordinated with the Special Mathematics subject module.




(a) (b)

Fig. 3. Example problems in the Circuits Theory subject module on the UBUCampus-e, coordinated with the Special Mathematics subject module.

## RESULTS

An evaluation of the pilot experiment was conducted during the 2004-2005 academic year through a survey of 3rd year electronic engineering students following the Mathematical Methods subject module. The questionnaire consisted of a set of 12 statements on which the student had to express their degree of agreement or disagreement, on a scale from 1 (completely disagree) to 5 (fully agree). Table 2 presents the survey results.

Table 2. Results of the survey of students following the Mathematical Methods optional subject module. Modal responses and average values

| nº | Statement | 5 Strongly agree | 4 Agree | 3 Neutral | 2 Disagree | 1 Strongly disagree | Mean |
|---|---|---|---|---|---|---|---|
| 1 | I prefer to learn each subject module separately, without anyone connecting up the subject modules. | | | 17% | 17% | 67% | 1.5 |
| 2 | I have enrolled on the subject module with no other interest than passing another module. | | | 50% | 50% | | 2.5 |
| 3 | I found the subject module of | | 83% | 17% | | | 3.8 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | interest because it helped me to see the relations between mathematics and other subjects. | | | | | | |
| 4 | A class that deals with problems related to other subject modules is of more interest to me than a class that deals only with mathematical problems. | | 100% | | | | 4.0 |
| 5 | I think I have understood more through working with problems related to other subject matters than through problems that are not related to those relations | | 80% | 20% | | | 3.8 |
| 6 | I have had difficulty relating the knowledge from each of the different subject modules. | | 33% | 33% | 17% | 17% | 2.8 |
| 7 | The practical classes on problems related to other subject modules give me worse information because I am unable to understand the relations that are set out. | 17% | | 50% | 33% | | 3.0 |
| 8 | As a teaching method, the practical mathematics classes using computers are more effective than the traditional practical classes. | 33% | 17% | 33% | 17% | | 3.7 |
| 9 | As a teaching method, the practical mathematics classes using computers are more agreeable than the traditional practical classes. | 50% | 33% | 17% | | | 4.3 |
| 10 | I have improved my analytical ability (to understand parts of the problem so as to understand the complete problem) thanks to the way in which the problems are presented. | | 50% | 50% | | | 3.5 |
| 11 | I have been able to test the validity of my earlier knowledge thanks to the way in which the problems are presented. | | 67% | 33% | | | 3.7 |
| 12 | My knowledge on other subject modules has increased thanks to the work undertaken in this one on mathematics. | | 17% | 67% | 17% | | 3.0 |

A number of conclusions may be drawn from these results. In the first place, the students are interested in learning about the interrelated subject modules (Question 1). Furthermore, the students think that they have gained better results in their learning (knowledge plus abilities) if they are presented with problems that relate mathematics with other subject modules. (Questions 5, 10 and 11).

Furthermore, the problems presented to the students reinforce the learning of mathematics perhaps more so than the subject modules with which the mathematics is related.

Finally, the students consider that the mathematics classes would be more agreeable and interesting if they presented problems that related mathematics with other subject modules and in addition if they were allowed to use modern methods of calculation such as computers. (Questions 4 and 9, which in fact score very highly: 4 and 4.3 over 5, respectively). Learning is more agreeable for them with a computer (Question 9), although this does not ensure that it will be more effective.

In general, the students express a considerable degree of agreement with the experiment that lets them see the way in which mathematics relates to other engineering subjects (Question 3), and they also recognise that they have learnt more from the engineering subject modules thanks to their work on the Mathematics module (Question 12).

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Making use of the UBU e-Campus tools for tracking student use of the materials, a study was conducted on the extent to which the joint notes were used. The notes were designed for the experiment and were introduced on the 'Circuits Theory and Special Mathematics' subject modules that formed part of the 2nd year course on Electronic Engineering, over the 2006/07 and 2007/08 academic years. The results are presented in Tables 3 and 4.

Table 3. Use of course notes for subject module 1: Circuits Theory; and subject module 2: Special Mathematics. Sections 1 and 2, as consulted on the UBUCampus-e Academic Years 2006/07 and 2007/08.

| Academic Year | % of students that have consulted subject module 1-Section 1 | Indicative measure of satisfaction with the notes for subject module 1-Section 1 | % of students that have consulted the joint notes for subject module 2-Section 1 | Indicative measure of satisfaction with the notes for subject module 2-Section 1 | % of students that have consulted the joint notes for subject module 1-Section 2 | Indicative measure of satisfaction with the notes for subject module 1-Section 2 | % of students that have consulted the joint notes for subject module 2-Section 2 | Indicative measure of satisfaction with the notes for subject module 2-Section 2 |
|---|---|---|---|---|---|---|---|---|
| 2006/07 | 99% | 3.9 | 99% | 4.1 | 99% | 4 | 99% | 4.3 |
| 2007/08 | 89% | 3.7 | 97% | 3.9 | 88% | 4.1 | 96% | 4 |

Table 4. Use of course notes for subject module 1: Circuits Theory; and subject module 2: Special Mathematics. Sections 3 and 4, as consulted on the UBUCampus-e Academic Years 2006/07 and 2007/08.

| Academic Year | % of students that have consulted subject module 1-Section 3 | Indicative measure of satisfaction with the notes for subject module 1-Section 3 | % of students that have consulted the joint notes for subject module 2-Section 3 | Indicative measure of satisfaction with the notes for subject module 2-Section 3 | % of students that have consulted the joint notes for subject module 1-Section 4 | Indicative measure of satisfaction with the notes for subject module 1-Section 4 | % of students that have consulted the joint notes for subject module 2-Section 4 | Indicative measure of satisfaction with the notes for subject module 2-Section 4 |
|---|---|---|---|---|---|---|---|---|
| 2006/07 | 87% | 4 | 99% | 3.8 | 76% | 4.1 | 99% | 4.3 |
| 2007/08 | 73% | 4.2 | 86% | 4 | 68% | 4 | 90% | 4.1 |

The following conclusions may be drawn from looking over the figures in the latter tables. Without doubt, students consider that the joint notes are useful, as the majority use them and in all cases rate them highly.

The drop that is observable in consultation rates over 2006/07 with respect to 2007/08 may possibly because students were already familiar with the notes from the earlier year, had photocopied and circulated them and consulted them less place over Internet. The differences in percentage consultations for one subject module with respect to another might be because Special Mathematics is a one semester module whereas Circuits Theory is over two semesters.

With respect to the ratings, they are slightly higher in the subject module on Special Mathematics, possibly because learning about topics related to mathematics stimulates greater interest in students than using the same ones that are inevitably related to Circuits Theory. The lowest figure in the consultations on Section 4 on Circuits Theory is due, perhaps, to the final data being definitively recorded before the course has come to an end and because it is the last topic that is taught.

## CONCLUSIONS

A study has been performed on the integration of engineering and mathematics knowledge in the teaching of Mechanical Engineering and Electronic Engineering, at the *Escuela Politécnica Superior* of Burgos University. A trial experiment was conducted on the combined teaching of Mathematics and Circuits Theory study modules on the 2nd year of a course leading to a degree in Electronic Engineering. This teaching is carried out through a combination of taught classes and a virtual teaching/learning platform, the UBUCampus-e. The results of the work were evaluated through perception questionnaires for the students and through the UBUCampus-e tracking system. In the first place, the difficulty of convincing the teachers to deepen their interdisciplinary knowledge, the need to integrate knowledge and the study of parallel knowledge between study modules may be pointed out as some of the principal difficulties encountered in the experiment. Also, worth mentioning is the considerable preparatory work by teachers from different knowledge areas to prepare the teaching materials and to use shared terminology and conventions. The teachers had a need for additional knowledge to guide a class when dealing with concepts from other study

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modules, in order to express their knowledge by directly establishing the relation with other subject matter from the same qualification. The main benefits are, firstly, the improvement in communication and coordinated work routines between teachers from different areas and departments, with the common aim of improving learning outcomes. Secondly, both teachers and students have been absorbing a better global, integrated vision from the materials in the same study plan. Thirdly, the fact that the students show interest in the question of joint notes that integrate knowledge and that the majority consult them when studying the relevant subject modules. Finally, this type of teaching helps students to develop a better overall capacity to analyse engineering problems in depth.


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