A Problem-Based Learning Approach to Numerical Methods: Interface Behaviour in Glass FRP Strips Bonded to Reinforced Concrete Beams

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Index Terms: Numerical Methods; Problem-Based Learning; Glass Fiber Reinforced Polymer; Bond Stress; Reinforced Concrete.

Abstract

A case of Numerical Methods using a Problem-Based Learning (PBL) approach aimed at Civil Engineering graduation students is presented: the interface behaviour in glass fiber reinforced polymer (FRP) strips bonded to reinforced concrete (RC) beams.

When a FRP plate is externally bonded to concrete, the interface FRP-concrete becomes a very important issue. The physical understanding of the interface is developed by various authors that proposed some bond-slip relations capable to explain the debonding phenomena. Thus, simplified models for simulating FRP-to-concrete interface behaviour are usually applied to evaluate and calibrate these empirical relations. Among many relations, there is known that the exponential bond-slip laws are the relations most appropriate to describe the nonlinear behaviour of the FRP-concrete interface [1], but these relations aggravate numerical difficulties.

A case problem [2] is treated under realistic conditions, with the calculations performed considering the desired precision level. For Civil Engineering graduation students, the focus is on the fundamentals, the methodology of Numerical Methods, and their limitations. Treatment and assessment of numerical issues are proposed [3, 4] in a way both simple and instructive, featuring: analytical *versus* numerical development; boundary-value definition and its impact; numerical integration facing infinity discontinuity; software limitations and coding empowerment; numerical condition of exponential function and error propagation; perspective on results.

1) Introduction

This communication describes a case study on Numerical Methods, aiming differential equations and integration principles, illustrating a simple numerical problem specifically built to fulfil Civil Engineering graduation curricula. A Problem-Based Learning (PBL) approach addresses a specific case on Civil Engineering, the case of the interface behaviour in glass fiber reinforced polymer (FRP) strips bonded to reinforced concrete (RC) beams [2], in an empowering way that matches student computational competences.

For Civil Engineering graduation students, the focus is on Numerical Methods, namely in the concepts supporting the treatment of ordinary differential equations (ODE), in the numerical integration methodologies, and in the compulsory adjustments due to its limitations. The referred case problem [2] is treated using the usual path of the Engineering problem resolution: the case problem is described and mathematically modelled, it is solved analytically and numerically, and results are analysed. The problem is viewed in detail, realistic conditions are assumed, and calculations precision level is considered. A tutorial guiding is proposed to support the reasoning, known by the tutor, but to be fully experienced by the students.

Treatment and assessment of numerical issues are proposed [3, 4], using a tutorial approach later described, and it features: an analytical approach that reveals itself insufficient, compelling the numerical development; the impact of boundary-values definition is illustrated, and students pay attention to its significance; the numerical integration faces infinity discontinuity which reveals software limitations, thus there is necessity of develop a simple integration code; on an instructive way, students are empowered to overpass those mathematical and numerical limitations, and results are analysed; the numerical condition of

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the exponential function and error propagation are considered, also as several extensions of the open-ended problem.

A Civil Engineer should know and apply basic sciences (Chemistry, Geology, Mathematics, Physics, among others) in a way to state and evaluate possible solutions for a problem situation, and in the Engineering education process one should take into account: several difficulties on basic sciences knowledge, also revealed by students in pre-university education; also, a general and relative marginalization of the basic sciences; lack on competences on problem treatment, in special when various and sequential steps are needed to achieve a solution; non-effectiveness on reasoning, on build strategies to relate and apply concepts belonging to several areas of science, or even to make decisions critically.

Attempting on that, competences related to problem-solving should be developed in the early stages of Engineering curricula, deepening further those competences through the presentation of complex problems, on a diffuse and uncertain framework, in a manner to create the necessity on students to build strategies that could address effectively the problem, and that can be systematically used when facing new problems. Engineering education should be directed to solve problems retrieved from complex situations, even from Master or PhD. themes, and adequately consider the possible and reasonable solutions. Our perspective is to use Numerical Methods issues and the competences needed on its effective implementation, and they can be developed through a PBL approach conjugating means and goal subjects. Numerical Methods knowledge and computational implementation competences constitute essential Civil Engineer added-value, considering that he must to apply mathematical basics to model the problem situation, to solve it numerically when the analytical approach is insufficient, to state and to analyse solutions, or he must to simulate the real problem or to optimize it.

The process of PBL is centred on students, as Bologna principles claim, and student is responsible for its active participation, not only on an individual way of learning, but also to aid the learning of other students committed on the same problem, *i.e.*, the tutorial group. PBL approach should consider real and professional situations and it requires: situation comprehension and identification of the important issues in the problem; understanding of the basic mechanisms behind these issues; several discontinuities on knowledge and understanding that may require additional studies or research; sharing knowledge and information with other students on tutorial group; tasks definition and to prioritize them, in relation to those learning issues; perspective on results and stating possible solutions.

The present Civil Engineering case considers a GFRP externally bonded to concrete, within the physical understanding of the interface needs to be developed. Only with deep comprehension we are capable to model the debonding phenomena by predicting and avowing this kind of premature collapse of reinforced concrete structures. For that, several bond-slip relations capable to explain the debonding phenomena are proposed, and simplified models for simulating FRP-to-concrete interface behaviour are usually applied. Among many relations, it is known that the exponential bond-slip laws are the most appropriate relations to describe the nonlinear behaviour of the FRP-concrete interface [1], but these relations aggravate numerical difficulties. Obviously, a linear or bi-linear bond-slip law conducts to a mathematical problem much easier to solve when compared with other laws and this is the main reason of these laws popularity. However, it is well known that linear and bi-linear laws do not accurately estimate the real behaviour of the interface between FRP-concrete, conducting to errors that might be more or less significant, depending on the framework of the study where Civil Engineers are involved.

Facing the described situation, the tutor's role is centred onto motivate student‘s learning experience, to assist to the identification of key issues, to monitor group discussion, to supervise tasks achievement, to assess results and solutions proposed by the students, or even to hint possible extensions.

The structure of this paper is as follows: a case study is presented, a description of several problematic issues occurring on its treatment is developed, and several mathematical formulations are proposed to model the specific problem; compulsory, beyond the analytical approach, some numerical methodologies aimed at Civil Engineering under-graduation students, i.e., Bologna first cycle students, are deepened and illustrated; results are analysed and learning extensions are proposed; also, as the PBL process states, a tutorial approach is described in a synoptic way; then, we present the main conclusions in a final section.

2) Description and modelling of the case

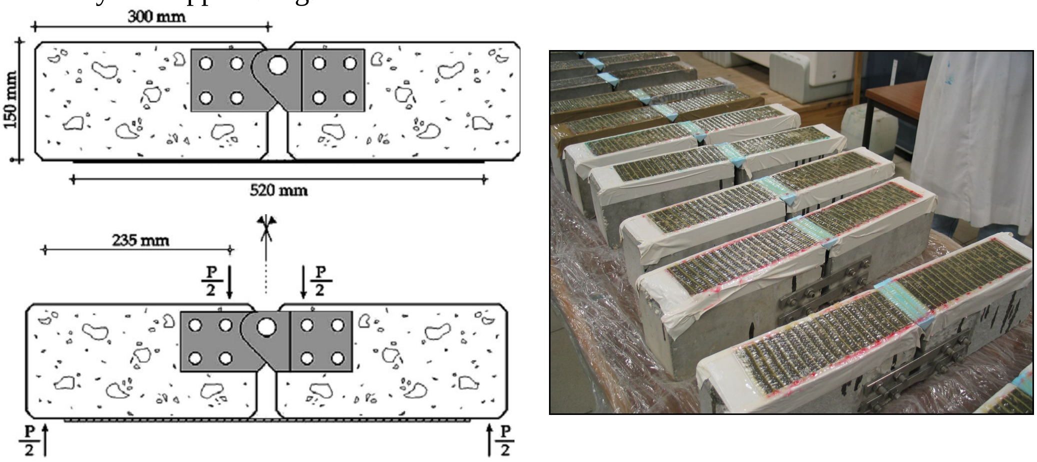
The case problem emulates one real and professional life situation, and it is selected and designed to require from students the acquisition of critical knowledge, problem-solving competences, learning strategies and team work skills. In the case, a complex and problematic situation is presented in a manner that its analytical resolution is insufficient. Then, the students must appeal to numerical treatment and perform their calculations with computational support.

For Civil Engineering graduation students, the focus is on the basic concepts behind the methodology of Numerical Methods, allowing: to gain a fundamental understanding of the importance of computers; to study the system sensitivity and behaviour, as Numerical Methods allow the computer to assume the computational burden; to code procedures that can illustrate the power and the limitations of computers; to analyse results and state possible solutions. This way, Numerical Methods can enhance problem-solving skills.

II.1 Problem description and statement

This short overview is closed to Aiello *et al.* [5] work, where studies similar to those reported hereafter are also described. The authors tested 90-cm-long plain concrete beam specimens reinforced with externally bonded wet-laid GFRP sheets. The beams were pre-cracked and subjected to four different environmental conditions including elevated temperature/dry, and freeze/thaw cycles. Debonding of the GFRP sheet from the concrete was the dominant failure mode and the ultimate strength of the GFRP reinforced beams reduced for specimens subjected to cycles of dry freezing and wet thawing. A downward trend in effective bond length with conditioning time was recorded.

The current study was based on several RC specimens externally reinforced with GFRP. These specimens were tested to study the time evolution of bond between concrete and GFRP under environmental aggression. However, and for mathematical propose, the environmental effects are negligible. Geometrically, the specimens are formed by two independent concrete blocks connected through a stainless steel hinge device, with dimensions as shown in Fig. 1a. The RC beams were externally reinforced with two layered GFRP strips bonded by the wet lay up technique to the tension side surface of the beams. The GFRP reinforcement was 1.3 mm thick, 520 mm long and 80 mm wide. GFRP strips were made of unidirectional SEH51/Tyfo GFRP fabric, with sparse Kevlar fibers orthogonal to the glass fiber. The resin used was Tyfo S epoxy as advised by the supplier. Fig. 1b shows the beams after conclusion of the GFRP reinforcement.



(a) (b)

Fig. 1 (a) Geometry of the specimens submitted to 4 points bending tests.
(b) Beam specimens after bonding of GFRP external reinforcement. [2]

The supplier indicated the following values for the properties of Tyfo S epoxy, obtained in accordance with the appropriate ASTM standards: Young modulus 3.18 GPa, tensile strength 72.4 MPa, ultimate strain 5.0% and a transition temperature of 82ºC.

Flat standard GFRP coupons were tested on a Zwick machine, at a rate of 2 mm/min, at the onset of artificial aging. The data generated led to average Young modulus of 20.4 GPa, tensile strength of 500 MPa and an ultimate strain of 2.5%. Matrix contribution to the strength of the GFRP is negligible when compared with the glass fibre, but higher than that of concrete (with a tensile strength of 2.7 MPa), a fact that is essential for transfer of load through bond.

The specimens were instrumented with strain gauges bonded to the GFRP strip, spaced at intervals of 40 mm to measure the strain distribution along the GFRP laminate at different loading levels. The strains were continuously recorded using a data-logger connected to a PC. Two LVDT transducers were used: one at the centre of the concrete block and the other at a distance of either 50 or 90 mm [6].

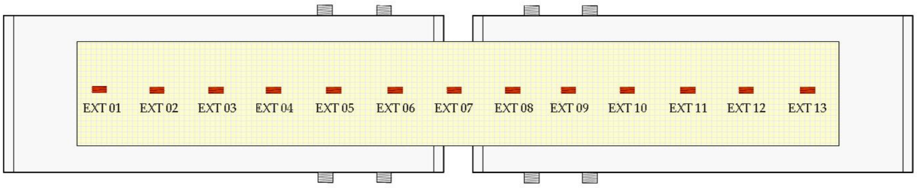


Fig. 2 Sketch of location of strain gauges to measure strains in four point bending tests. [2]

Bending tests were performed as displayed in ***Fig. 3***. The loading was applied via an actuator and its magnitude was measured by two pressure cells placed under the supports of each specimen and recorded in the data-logger. The end results of the bending tests are typically represented in ***Fig. 4*** where the failure of a reference beam is shown. However, some differences were detected for some specimens, though, in some beams examines the surface separation since the debonding mechanism was not always the same. In fact, some beams, revealed a debonding by the interface between GFRP-concrete and others specimens showed a cohesive rupture in the concrete due to concrete tensile failure and, in these cases, a thick layer of concrete can be seen adherent to the GFRP strip after failure.

Fig. 3 Set up for application.

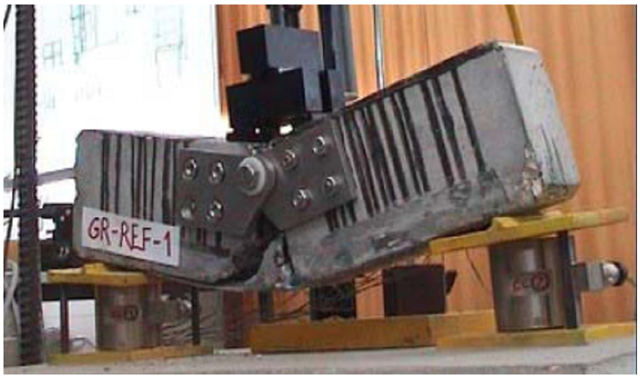


Fig. 4 Failure of a reference beam.

II.2 Problem modelling

A case problem [2] is treated under realistic conditions, with the calculations performed considering the desired precision level.

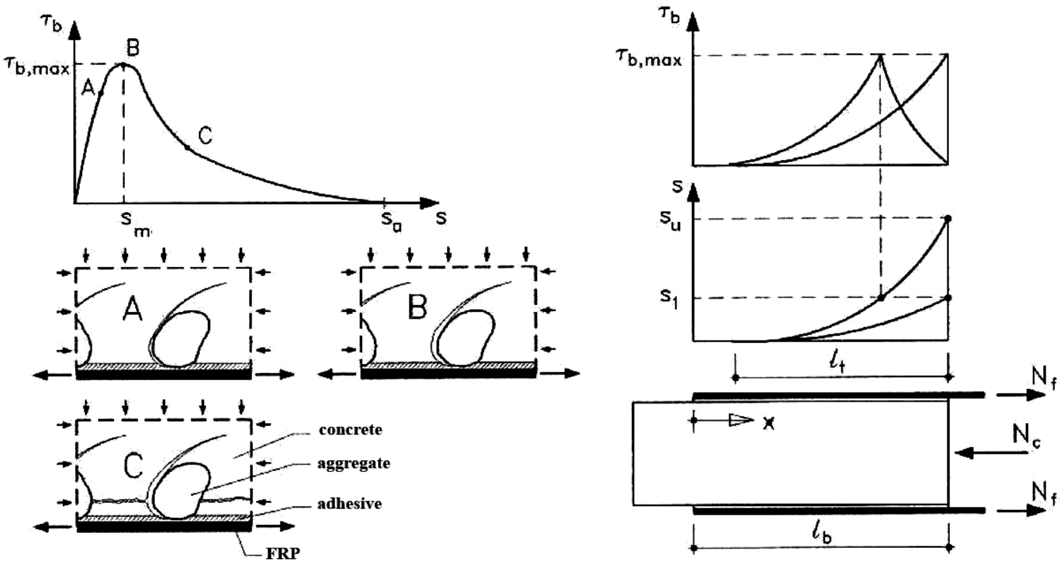


Fig. 5 Mechanism and bond experimental behaviour. Based in [7].

As it was said early, the most approximate behaviour of the interface between a FRP and concrete is represented on the left side of ***Fig. 5*** where is adopted an exponential bond-slip law. Thus, the physical phenomena may be described as follow: closed to the origin, the bond reveals a linear behaviour due to the linear statement of both materials; as the load increases the development of micro-cracking in concrete near the interface FRP-concrete also increases. At that stage (point A), a previous treatment of the concrete superficies, for instance with jet sanding, becomes an important issue. The concrete micro-cracking occurs between the aggregates and cement leading to a non-linear behaviour of the bond (with a good treatment of the superficies it is possible that aggregate stay glued to the FRP). The micro-cracking propagation reaches is limit when the limit of the bond shear stress is achieved (point B). The bond-slip descendent curve (point C), correspond to the horizontal development cracking between aggregates closed to the interface FRP-concrete. Finally, when the FRP debonding occurs the shear stress is null. This behaviour is the main reason for the assumption of exponential laws. Also, in nowadays, with powerful computers these exponential laws become more common in some commercial codes, predicting with more precision the bond stress development through interface FRP-concrete. It is physical understandable that during the pre peak phase (pre point B) the linear bond slip laws could represent very well the bond stress distribution. However, during the phase pos peak (pos point B) it is recommended to use exponential laws to simulate the development and the assumptions of theories related with non-linear fracture mechanics. In fact, it is more often to find those theories implemented in different commercial codes of the speciality [2].

The typical behaviour of a simple or double shear test in order to bond stress and slip along the FRP is shown in the right side of ***Fig. 5***.

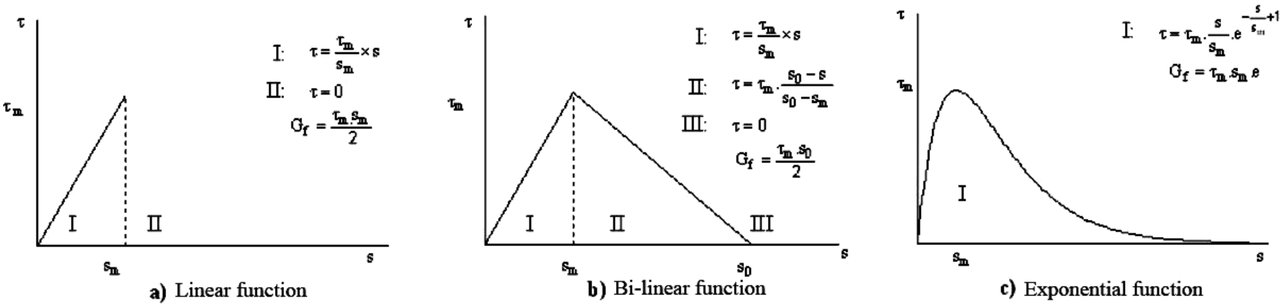


Fig. 6 Common bond-slip laws found in literature. Based in [7].

The simplest way for simulating FRP-concrete interface behaviour consists in assuming uncoupled models for shear and normal stresses (***Fig. 7***) [8].

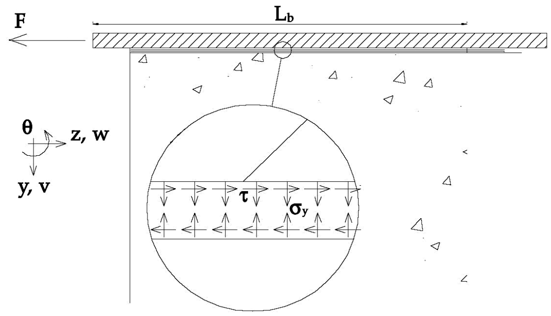


Fig. 7. FRP-concrete joint [8].

The equations ***Eq.1*** and ***Eq.2*** are the equations that traduce the equilibrium the compatibility conditions in direction *z* stated as follows:

Eq.1
$$\frac{\partial \sigma_z}{\partial z} - \frac{\tau(s)}{t_f} = 0$$

Eq.2
$$\varepsilon_f = \frac{ds}{dz}$$

The equations ***Eq.1*** and ***Eq.2*** lead to the follow differential equation in terms of shear stress, $\tau(s)$, and interface slip, *s*:

Eq.3
$$\frac{d^2 s}{dz^2} - \frac{\tau(s)}{E_f \cdot t_f} = 0$$

3) Compulsory numerical methods

Several authors studied the numerical modelling of the FRP-concrete interface exiting now some proposals in this area that relates the bond stress and the slip between FRP and concrete (see ***Fig. 6***). These relations are highly dependent of the concrete strength and it can be experimental proved by higher bond stresses development when FRP is glued to higher performance concrete. Thus, it can be assumed that the slip between FRP and an high performance concrete is only due to the FRP extensions and ***Eq.6*** is due only to the FRP extension that were obtained by an experimental way as described in [2].

III.1 Analytical approach

Assuming a bi-linear bond-slip law for τ(s) as represented in ***Fig. 3.6.b***, two cases may occur, but for the complexity target to student it only be analysed one case: if $s \le s_{el}$ throughout all the bonded length (elastic behaviour) the following exponential solution can be derived [8]:

Eq.4 $$\frac{d^2 s}{dz^2} - \alpha_{el}^2 \cdot s = 0 \quad \text{considering} \quad \alpha_{el}^2 = \frac{k_{el}}{E_f \cdot t_f}$$

Eq.5 $$s = A \cdot \cosh(\alpha \cdot z) + B \cdot \sinh(\alpha \cdot z)$$

Being A and B two integration constants obtained by limit conditions established in ***Eq.6***, the final solution of the differential equation in ***Eq.3*** is ***Eq.8***.

Eq.6 $$\begin{cases} z = 0 \\ \sigma_f = \dfrac{P}{b_f \cdot t_f} \end{cases} \Rightarrow \frac{ds}{dz} = \frac{P}{E_f \cdot b_f \cdot t_f} \quad \text{and} \quad \begin{cases} z = L \\ \sigma_f = 0 \end{cases} \Rightarrow \frac{ds}{dz} = 0$$

Eq.7 $$A = -\frac{P}{\alpha_{el} \cdot E_f \cdot b_f \cdot t_f} \cdot \frac{\cosh(\alpha \cdot L)}{\sinh(\alpha \cdot L)} \quad \text{and} \quad B = \frac{P}{\alpha_{el} \cdot E_f \cdot b_f \cdot t_f}$$

Eq.8 $$s = \frac{\tau}{k_{el}} = \frac{\alpha_{el}}{k_{el}} \cdot \frac{P}{b_f} \cdot \frac{\cosh[\alpha_{el} \cdot (L - z)]}{\sinh(\alpha_{el} \cdot L)}$$

Using ***Eq.8*** the bond stress of the FRP-concrete interface may be written as follow:

Eq.9 $$\tau = \alpha_{el} \cdot \frac{P}{b_f} \cdot \frac{\cosh[\alpha_{el} \cdot (L - z)]}{\sinh(\alpha_{el} \cdot L)}$$

Where α_{el} is a parameter given by ***Eq.10***,

Eq.10 $$\alpha_{el} = \sqrt{\frac{k_{el}}{E_f \cdot t_f}}$$

Thus, the maximum load corresponding to elastic slip of the interface is given by load P_{el} witch is obtained as follow:

Eq.11 $$P_{el} = \frac{\tau_{max}}{\alpha_{el}} \cdot b_f \cdot \tanh(\alpha_{el} \cdot L_b)$$

III.2 Numerical approach: introduction

Assuming an exponential form to relation *τ(s)*, such as

Eq.12 $$\tau(s) = \tau_m \cdot \frac{s}{S_m} \cdot \exp\left(1 - \cdot \frac{s}{S_m}\right)$$

an analytical approach to solve the second order nonlinear ODE presented in ***Eq.3***, like the approach described in the last subsection, is no longer suitable. Thus, treatment and assessment of numerical issues are proposed [3, 4], in a way both simple and instructive. A treatment proposed to solve the second order nonlinear ODE is, as usual, supposing an intermediate function *p(s)*, varying only in *s*:

Eq.13

$$p(s) = \frac{\mathrm{ds}}{\mathrm{dz}} \quad (= \varepsilon_f)$$

This intermediate function *p(s)* can be physically interpreted as a fiber extension measure, or the relative fiber deformation, ε_f. Applying it onto ***Eq.3***, one first analytical integration results on a square root relation,

Eq.14

$$p(s) = \frac{\mathrm{ds}}{\mathrm{dz}} = \sqrt{K_{\mathrm{int}\,1} - \frac{2\tau_{\mathrm{m}}}{\mathrm{E}_f \mathrm{t}_f} \cdot \exp\left(1 - \frac{\mathrm{s}}{\mathrm{S}_m}\right) . (s + S_m)}$$

where K_{int1} represents a constant term, resulting from integration.

But the second integration on the ODE, following the equality

Eq.15

$$\mathrm{dz} = \frac{\mathrm{ds}}{p(s)} = \frac{\mathrm{ds}}{\sqrt{K_{\mathrm{int}\,1} - \frac{2\tau_{\mathrm{m}}}{\mathrm{E}_f \mathrm{t}_f} \cdot \exp\left(1 - \frac{\mathrm{s}}{\mathrm{S}_m}\right) . (s + S_m)}}$$

results on a compulsory numerical treatment of the second member integral,

Eq.16

$$\mathrm{z}_{\mathrm{L}} - \mathrm{z}_0 = \int \frac{\mathrm{ds}}{\sqrt{K_{\mathrm{int}\,1} - \frac{2\tau_{\mathrm{m}}}{\mathrm{E}_f \mathrm{t}_f} \cdot \exp\left(1 - \frac{\mathrm{s}}{\mathrm{S}_m}\right) . (s + S_m)}} + K_{\mathrm{int}\,2}$$

where K_{int2} represents the integration constant term.

Teaching objectives are considering the treatment of a nonlinear second order ODE, whom allows comparison between different relations to model *τ(s)*, enlarging student competences on ODE calculus, research by the student himself, and team work. Also, the issue of stating boundary-values, along with the assessment of its impact on integration constants, must be evaluated focusing its physical significance.

III.3 Numerical approach: boundary-values and its impact

In a manner to treat numerically the integral in ***Eq.16***, one must evaluate accurately the constant terms K_{int1} and K_{int2}, using the boundary-values. However, applying the usual boundary condition at **z = L** as zero value for fiber extension or relative deformation, ε_f, as stated in ***Eq.6***, and due to the necessity of obtaining a specific value for slip, *s*, when one considers it zero the integrand function 1/p(s) presents an infinity discontinuous value.

An alternative boundary-value, and numeric comparison could easily be made, is stating a constant and non-zero value for slip, *s*, in the boundary condition at **z = L** as follows:

Eq.17

$$\begin{cases} \mathrm{z} = 0 \\ \sigma_f = \dfrac{\mathrm{P}}{\mathrm{b}_f \cdot \mathrm{t}_f} \end{cases} \Rightarrow \frac{\mathrm{ds}}{\mathrm{dz}} = \frac{\mathrm{P}}{\mathrm{E}_f \cdot \mathrm{b}_f \cdot \mathrm{t}_f} \quad \text{and} \quad \begin{cases} \mathrm{z} = \mathrm{L} \\ \sigma_f = 0 \end{cases} \Rightarrow \left(\frac{\mathrm{ds}}{\mathrm{dz}} = 0 = p(L)\right), \quad \mathrm{s} = Konst\,(\neq 0)$$

Thus, teaching objectives are aiming alternative boundary-values within its physical significance, and its numerical treatment allows students to develop specific competences on a framework usually not considered in Numerical Methods curriculum: we mean the numerical integration facing infinity discontinuity, and it can be introduced here through a defying situation that has to be overpass, *i.e.*, solved.

III.4 Numerical approach: integration facing infinity discontinuity

This numerical issue, due to infinity discontinuity, results on severe error onto the integral estimation of ***Eq.16***, even when one corrects the first interval of the applied numerical integration method, because the main point that student has to deal is the infinity value occurring there.

As described above, lack of accuracy on evaluation of integration constants causes several propagating errors, successively, in the evaluation of the square root in ***Eq.14*** that defines *p(s)*, in the evaluation of its inverse value 1/*p(s)*, and finally in the integral stated as outcome in ***Eq.16***.

Teaching objectives are targeting the most common approaches for numerical integration, like trapezoidal rule or Simpson's rules, assuming adjusts on integration intervals aiming the comparison to experimental values, working out the number of integration intervals and assessing the respective evolution of relative error, or even extending research to numerical schemes like Romberg integration that allows to attain this kind of problem efficiently. However, the referred approximation procedures are not applicable if the zero boundary-value on slip *s* is exactly assumed, due to the fact that this defying problem can not be treated by usually available numerical tools, and one may suggest the development of some numerical integration code as alternative.

III.5 Numerical approach: software limitations and coding empowerment

Software characteristics are illustrated through this numerical issue, revealing its potential and operation capabilities when considering the main subjects of a software package, but also revealing its limitations when specific issues like this one that we are presenting.

As alternative to commercial packages (for example: Matlab, Mathematica, even a spreadsheet like Excel) utilization, one may advise the use of some code language (for example: Fortran, C) as a complementary tool, or one may develop some code inside the commercial package environments like those referred.

Coding empowerment is an achievable point to the student, when in face of a problem he can surpass the use of standard software. Specifically for this case, one may recommend the simple use of a rectangular integration approach, that applies the middle point in each integration interval, in spite of numerical schemes like trapezoidal rule, that are using interval extreme points. This way, one can avoid the evaluation of the function on zero point for the first interval, avoiding also the infinity discontinuity.

The main teaching objective is the intelligent utilization of the available tools, comparing and selecting between commercial packages and code development, knowing its limitations and requirements, and evaluating the related trade-offs. Also, student empowerment is targeted when the code versatility allows the student to not be software-limited, and this point gives student additional motivation and insight. Notwithstanding this type of achievement, there are some numerical issues deeply ingrained in this case and that must be considered, such as the numerical condition of exponential function and error propagation.

III.6 Numerical approach: error propagation

The numerical condition of a function allows the analysis of error propagation through functional relationships, and a value greater than one represents the growth between the relative error in the function value and the relative error in its argument.

The exponential function presented in ***Eq.12***, that defines the relation *τ(s)*, always has a positive exponent, and this implies a numerical condition greater than one. The meaning is an error propagation of relative error in slip, *s*, through the evaluation of *τ(s)*. A careful analysis could reveal critical issues, as the known subtractive cancellation error when integration constants are evaluated, or even the evaluation of an inverse of a function near zero value.

The main teaching objectives are related to the concepts of numerical condition, error propagation and stability, in particular, to recognize how computer arithmetic can introduce and amplify round-off errors in calculations. Extensions can be performed, research on this subject developed, but it may be addressed to the final results expected, in comparison with the experimental values.

4) Perspective on results

The case insight is deepened applying the described numerical approach, intermediate answers are analysed and selected, possible solutions are stated on an open-ended framework, and perspective on results is extended. Also, it must be noted that students should be aware that the solutions obtained from the model must be validated against the real data.

Concerning intermediate answers, some subjects may require additional research, namely:

- The supposition of an intermediate function *p(s)* varying only in *s*;
- The boundary-values definition, and its physical significance;

- The accurate evaluation of the integration constant terms K_{int1} and K_{int2}, using the boundary-values;
- The numerical integration issue of facing infinity discontinuities;
- The stability subject, depending on error propagation and the numerical conditioning involved.

Also, some extensions focusing exposures that are causing failure on the interface between the resin and the concrete, may lead to additional studies in the following subjects:

- $s(z)$ values numerically calculated and its assessment through experimental values of $\tau(s)$;
- Temperature effects and aging associated with salt water;
- Effects of curing and plasticization of the epoxy resin;
- Requirements needed to gather statistical significance;
- Certification of computational models against experimental data.

Teaching objectives consider the comprehension and identification of the main issues of the problem, the situation understanding, the fill of discontinuities in knowledge of basic sciences, active team work, information and problem treatment, sequential steps definition aiming complex problems, computational implementation and application of Numerical Methods, build strategies and make decisions. As the proposed path is quite complex, the student needs some monitoring and supervising support, that can be performed by the tutor as the PBL process states.

5) Tutorial guiding

The tutor is not necessarily a specialist in Structural Civil Engineering, but he must be a motivator of the learning experience of the student. Also, the uncertainty and complexity that characterize the PBL approach are incorporating two kinds of benefits: uncertainty contrasts with the absolute knowledge; application of Numerical Methods to a complex problem are driving to strategic processes of solving problems.

The supervisor role supports software utilization, being commercial packages or programming languages, in a way that the problem is numerically solved and the importance of using computer is enhanced.

A tutorial approach is described in a synoptic way, with the main subjects described as follows [9]:

- To monitor tutorial group progress, ensuring that all members of the group will take an active role;
- To avoid "mini-lectures", but promoting the use of schemes, concept-maps, and summaries;
- To prepare information sharing sessions, questioning systematically individuals or group, assuring information availability, setting up seat arrangements, setting up guiding introductions;
- To inform of the objectives for PBL and its evaluation process;
- To supervise learning tasks, suggesting about issues that must be treated individually or in group, defining specific questions aiming each student individually, recommending the approach or methodology suitable in each case.

At the end of the resolution process, it must be reviewed, asking in a question like "*how did we do*?", and do not forgot that PBL cases are open-ended, so it is always necessary to provides a holistic insight to the case, as a situation with multiple facets.

6) Conclusions

A case of Numerical Methods using a Problem-Based Learning approach aimed at Civil Engineering graduation students is presented: the interface behaviour in glass fiber reinforced polymer (FRP) strips bonded to reinforced concrete beams.

Numerical Methods allows the intelligent utilization of computer, and the results obtained through that have to be compared with the real values originated from experimental work. It must be noted that the study of sensitivity and behaviour of the numerical system is facilitated as Numerical Methods allow the computer to assume the computational burden. In this manner, a stronger exposition of the relationship of problem implies the development of holistic reasoning and to fine-tune intuition.

Besides the empowering of Numerical Methods for learning to use computers, they present themselves as a tool to reinforce the understanding of Mathematics and to enhance problem-solving skills, due to the fact that these methods usually reduce higher mathematics to basic arithmetic an its operations. Also, the coding effort required to the improvement of characteristics like convergence, stability and accuracy can be creative, and improvements can be easy to apply, such the referred rectangular integration rule.

The knowledge of the basic theory underlying Numerical Methods supports the effective use of commercial packages, specifically when student realizes that various problems cannot be approached using these packages. Thus, in addition to software packages, the student may develop and use his own programs, and he gains a strong understanding of the importance of knowledge about computers and how to apply it.

The present Civil Engineering case considers a GFRP externally bonded to concrete, within the physical understanding of the interface needs to be developed, to avowing the premature collapse of reinforced concrete structures. Several bond-slip relations are proposed, and simplified models for simulating FRP-to-concrete interface behaviour are applied. It is known that the exponential bond-slip laws are the relations most appropriate to describe the nonlinear behaviour of the FRP-concrete interface, but these relations aggravate numerical difficulties, in special when compared to linear or bi-linear bond-slip laws, that conduct to a mathematical problem easier to solve. However, linear and bi-linear laws do not accurately estimate the real behaviour of the interface between FRP-concrete, conducting to errors more or less significant.

The case problem is treated under realistic conditions, where the calculations consider the desired precision level, the error propagation due to numerical conditioning and the procedure stability. For Civil Engineering graduation students, the focus is on the fundamentals, the methodology, and the limitations. of Numerical Methods. Treatment and assessment of numerical issues are proposed, in a way simple and instructive that features: analytical and numerical developments; impact of boundary-value definitions; numerical integration in face of infinity discontinuity; limitations on software compelling coding development; analysis on error propagation; perspective on results.

For Civil Engineering graduation students, the focus is on the basic concepts of Numerical Methods, allowing a fundamental understanding of the importance of computers, a study to the system sensitivity and behaviour, the illustration of power and limitations of computers, and finally to analyse results and state solutions. In this manner, Numerical methods can improve problem-solving competences.

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