Integrating Rigid Body Dynamics & Vibrations: An Introductory Course for Undergraduate Civil Engineers

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Course Description

Application of Newtonian and energy methods to model dynamic systems (particles and rigid bodies) with ordinary differential equations; solutions of models using analytical and numerical approaches; interpreting solutions; linear vibrations.

Traditionally advanced topics

- kinematics coverage that emphasizes direct differentiation of vector components to obtain velocity and acceleration relationships in Cartesian, polar, or path coordinate systems; and transformation of answers to the remaining two systems; *and*
- a parallel analysis of the same examples using free-body diagrams, conservation of energy approaches to derive the equations of motion; *and*
- an "equal-time" policy when introducing planar kinematics, working examples alternately with traditional vector approaches and then the geometric approach, *and*
- including vibrations material and examples (including eigenvalues and eigenvectors).

Student reactions

- "I finally am taking an engineering course where I feel like I can or will use this in the 'real-world'"
- "One of the best things was when you showed how this directly applies to a civil engineering problem" (referring to how to model civil systems using a masses and springs MDoF model)
- "The projects really helped to bring concepts to life"
- "The projects were the heart of my interest for the class. Without civil application I could care less about dynamics."

Highlights of Learning Objectives

Objective #1- Planar kinematics for particle motion:

- Use Cartesian, polar and path-coordinate kinematics to define the velocity and acceleration components of a point in motion.
- Mathematically differentiate functions of time and space coordinates to determine desired functional forms.

Objective #2 - Physical modeling of particle dynamics (1 DOF):

- Identify the fundamental components of structural systems into generalized lumped mass (inertia) M, stiffness K, damping C elements.
- Determine the degrees of freedom and/or the constraints present.
- Establish the equivalence of Kinetic and Potential (Strain) Energies in Conservative systems.
- Derive the fundamental equations governing the motion of lumped- systems in general plane motion.
- Fundamental knowledge of kinematics and kinetics of planar rigid body motion: rectilinear motion and rotational motion about a rigid axis.

Objective #3 - Mathematical Modeling of 1 DOF systems:

- Determine analytically the dynamic response of 1DOF systems described by the linear ODE and given initial conditions.
- Determine the free response to initial conditions and the dynamic response to Impulse and Step loads.
- Derive dynamic response to harmonic forcing functions and discuss the regimes of operation: below, close to, or above its natural frequency.
- Obtain the Frequency Response Function (FRF) for sustained periodic excitations and explain the effects of system parameters on response.

Objective #4 - Mathematical Modeling of 2 DOF systems:

- Derive the EOMS for 2- or M-DOF lumped parameter systems and linearize the EOMs about equilibrium or operating point.
- Determine analytically eigenvalues and eigenvectors for undamped MDOF systems. Explain concept of modal coordinates and mode shapes.
- Use the transformation to uncouple the EOMS in physical coordinates and determine (analytically) the free and forced response of both undamped and damped MDOF systems to arbitrary initial conditions, step and periodic loads.

Objective #5 - Planar Kinematics for Rigid Bodies:

- Learn and be able to use two-coordinate systems to define the velocity and acceleration of a point in plane motion.
- Develop general kinematic equations for planar motion of rigid bodies and systems of rigid bodies including planar mechanisms.

Objective #6 - Planar Kinetics for Rigid Bodies:

- Develop dynamic models for planar motion of rigid bodies using both Newtonian and work-energy approaches.
- Analysis and simulations for planar motion of rigid bodies.
- Develop models for two-degree of freedom planar-kinetics examples and for planar mechanisms from free-body diagrams.

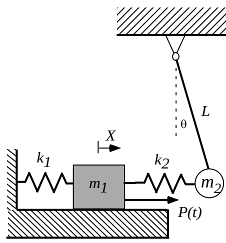
Objective #7 - Numerical Modeling of structural systems:

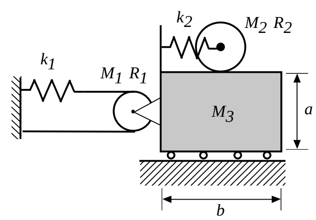
- Use computational software to solve linear and nonlinear algebraic and differential equations describing the motion of 1- or M-DOF systems.
- Apply knowledge gained in numerical methods to select appropriate numerical techniques with due consideration for time steps and procedures (algorithms) ensuring accurate, numerically stable, and cost efficient system response.
- Interpret numerical calculations (predictions) to explain system behavior, identify possible failure mechanisms due to excessive amplitudes of motion or reaction forces, etc.

Typical Exam Problems

Problems include typical rigid body dynamics problems, and single degree of freedom vibration problems found in any introductory textbook; ***AND*** problems such as:

Problem: Find the equation of motion for the system below left. Clearly and completely define your degrees of freedom (equilibrium or undeformed position? Total or relative motion? What is the positive direction?). Assume No Slip Conditions. Be sure to indicate any additional assumptions.





Problem: A pendulum-mass system is shown above right. Taking the horizontal translation of the mass, *X*, and the angle of the pendulum, θ, as your degrees of freedom ,

(a) Show that the Equation of Motion under small deformations is:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 L^2 \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 L \\ -k_2 L & (m_2 g L + k_2 L^2) \end{bmatrix} \begin{Bmatrix} X \\ \theta \end{Bmatrix} = \begin{Bmatrix} P(t) \\ 0 \end{Bmatrix}$$

(b) Find the mode shapes and corresponding frequencies of the system. Set up the linearized-eigenproblem equations symbolically first. The use the following values to solve:

$$m_1 = m_2 = 4 \text{ kg};\ \ k_1 = 2k_2 = 4 \text{ kg/m};\ \ L = 0.5\text{m};\ g = 9.81 \text{ m/s}^2$$

(c) What can you infer from the mode shapes regarding any general motion of the system? (State in words)

(d) Approximate the <u>**total**</u> response of this system using only the first mode if $P(t)$ is a harmonic load given by:

$$P(t) = 0.5 m_1 g \sin\left(\frac{3\omega_1}{4}\right)$$

ω_1 is the undamped natural frequency of the first mode.

Outcomes Addressed in MEEN 363

ABET Outcomes	**Evidence that Program Outcomes were Addressed**	***Assessment of Program Outcomes***
a. Apply basic math, science, & engineering	All lecture topics	In-class exercises, Individual Homework, Team Assignments, and Exams
d. Function in multi-discipline team	Teaming section in syllabus Team assignments: problem submission requirements	Team Assignments – in class exercises and homework problems
e. Formulate & solve CE/OE problems	All lecture topics	In-class exercises and suggested problems, Team Projects, and Exams
g. Communicate effectively	Team assignments: problem submission requirements	Team Assignments
l. Use computers to solve CE/OE problems	Team assignments	Individual Homework and Team Assignments

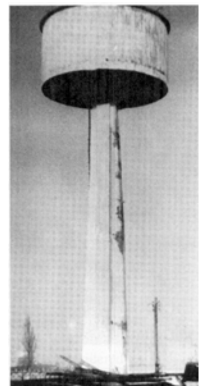
Typical CE Projects

A major course component is the use of computational projects to solve civil engineering dynamic problems.
Characteristics:

- Centered on different real civil engineering systems
- Designed to be solved by student teams, who are to act as consultants on the project posed.
- Teams required to evaluate alternatives and make a recommendation

Explicit Objectives:
(1) Tackle larger and more realistic civil engineering dynamics problem,
(2) Apply computational tools commonly used in solving dynamics problems for which a closed form solution does not exist,
(3) Exercise and evaluate critical thinking and communication skills.

#1 Project Goal and Objectives:



1. Determine dynamic properties of existing system
2. Evaluate the free vibration response
3. Evaluate the response under wind load
4. Suggest design change to either mass or stiffness properties so that the response meets the following response criteria:
   Peak response

#2 Project Goal and Objectives:

1. Determine equations of motion for the 3 models discussed, both linear and nonlinear.
2. Evaluate the free vibration response under a range of initial conditions
3. Evaluate:
   - Impact of linear vs. nonlinear models
   - Impact of particle vs. rigid body models

#3 Project Goal and Objectives:

1. Determine dynamic properties of existing system
2. Evaluate the frequency response of this structure by looking at response under harmonic loads
3. Determine the modal properties of the structure.
4. Determine response of structure under seismic loads
5. Evaluate the response of the structure. Parameters to be considered:
   - Peak displacement response
   - Peak interstory drift response
   - Peak base shear
6. Recommend design solution. Possible alternatives:
   - Introduction of braces at each story
   - Introduction of base isolation system

Starting from a realistic frame