The HELM Mathematics Learning and Assessment Regime for Engineering Students

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Abstract — *HELM (Helping Engineers Learn Mathematics) is a major three -year curriculum development project. This paper describes the HELM learning re sources with particular attention being paid to the engineering examples and case studies. The implementation of the HELM assessment regime and its use for both formative and summative assessment of engineering students learning mathematics are outlined. Finally the viability of implementing the HELM learning and assessment regime elsewhere is examined.*

Index Terms — *CAA, e -learning, assessment, mathematics, engineering undergraduates, HELM.*

INTRODUCTION

The United Kingdom government's Smith Inquiry [1], which reported in 2004, makes the following points:

- Mathematics is a major intellectual discipline in its own right, as well as providing vital underpinning for science and engineering.
- The UK continues in a situation of long term decline in the numbers of young people continuing to study mathematics post–16.
- Many believe there is a crisis in the teaching and learning of mathematics in UK schools (and therefore in UK Universities).

The Helping Engineers Learn Mathematics (HELM) project [2] has developed a strategy for addressing the practical difficulties currently encountered in the teaching of mathematics to engineering undergraduates. This comprises the production and dissemination of high quality extensive teaching and learning materials supported by a comprehensive CAA testing regime. At Loughborough University we have put in place an environment for learning mathematics that we believe will be attractive to the vast majority of undergraduate students, whatever their level and whatever their previous experience.

PRECURSOR TO THE HELM PROJECT

Historically the HELM project has developed its resource base on the earlier work of Loughborough's Open Learning Mathematics Project (OLMP) [3]. In 1997, funding was made available by the University for the OLMP to provide academic staff time and the part-time services of a learning technologist to develop learning materials.

The OLMP predated the trouble-free use of MathML and therefore the body of each question in the OLMP CAA regime was presented as a jpg image [4]. Following this, jpg images originated from a LaTeX file are used in newly developed questions. The image quality is further enhanced where necessary using a graphics application before being used in QM Perception [5]. With this approach already in place to produce consistent good quality images, we have maintained the same methodology for the development of new questions in QM Perception. The MathML approach was considered but, in view of the additional expertise required to develop questions and the need for users to have MathML enabled browsers, its use was not deemed to be ideal at present.

The success of the OLMP encouraged staff to seek funding to develop further this work resulting in the HELM project which is supported by a £250,000 grant from the Higher Education Funding Council for England – Fund for the Development of Teaching and Learning for the period Oct 2002-Sept 2005.

OVERVIEW OF THE HELM PROJECT

The HELM team consists of staff at Loughborough University and four consortium partners in other English universities: Hull, Reading, Sunderland and the University of Manchester Institute of Science and Technology (UMIST). The project aims to considerably enhance, extend and test Loughborough's original OLMP materials, in particular by the writing of many additional Workbooks and incorporating engineering exercises and case studies closely related to the mathematics presented, extending the question databanks, and promoting widespread trialling. The HELM project's output will consist of Workbooks, Computer Aided Learning (CAL) segments and a Computer Aided Assessment (CAA) Regime which is used to help 'drive' the student learning. (To view sample materials visit http://helm.lboro.ac.uk).

The Workbooks may be integrated into existing engineering degree programmes either by selecting isolated stand-alone units to complement other materials or by creating a complete scheme of work for a semester, a year or two years by selecting from the large set of Workbooks available. These may be used to support lectures or for independent learning, or a mixture.

CAL segments are provided for topics covered in typical UK first year undergraduate engineering mathematics courses, covered by the first 20 workbooks.

The banks of CAA questions developed by the HELM project presently contain around 5000 questions but it is anticipated that this number will rise to around 8000 on completion of the project. Original questions are held in Question Mark Perception version 2.5 format, these are being reviewed and transferred to QMP version 3.4, while new questions are being developed directly in version 3.4.

Nothing on this scale has been attempted before for free dissemination across the HE sector in England and Northern Ireland. The emphasis is on flexibility – the work can be undertaken as private study, distance learning, can be teacher-led, or a combination, according to the learning style and competence of the student and the approach of the particular lecturer.

HELM PROJECT WORKBOOKS

The main project materials are the Workbooks which are subdivided into typically four manageable Sections. As far as possible, each Section is designed to be a self-contained piece of work that can be attempted by the student in a few hours. In general, a whole Workbook represents for a student about 2 to 3 weeks' study at university. Each Section begins with statements of pre-requisites and the desired learning outcomes.

A Workbook Section consists of an introduction and the presentation of mathematical concepts, simply explained, interspersed with worked examples. The examples may be purely mathematical or within an engineering context (See Figure 1). A fully worked solution is provided immediately following the example (see Figure 2).

Example 5

The equation governing the buckling load P of a strut with one end fixed and the other end simply supported is given by $\tan \mu L = \mu L$ where $\mu = \sqrt{\dfrac{P}{EI}}$, L is the length of the strut and EI is the flexural rigidity of the strut. For safe design it is important the the load applied to the strut is less than the lowest buckling load. This equation has no exact solution and we must therefore use the method described in this Workbook to get the lowest P.

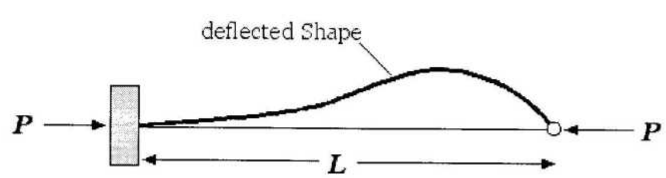
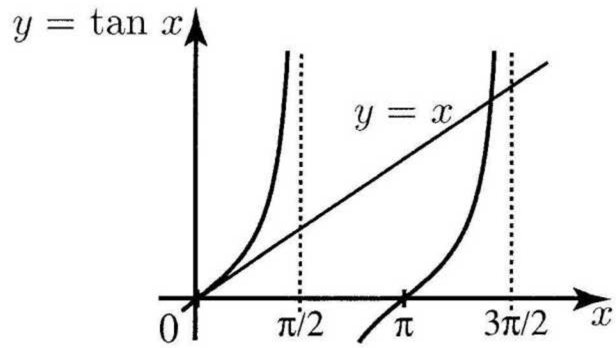


FIGURE 1
A TYPICAL ENGINEERING FOCUSED EXAMPLE

Solution

We let $\mu L = x$ and so we need to solve the equation $\tan x = x$. Before starting to apply the Newton-Raphson iteration we must first obtain an approximate solution by plotting graphs of $y = \tan x$ and $y = x$ using the same axes.



From the graph it can be seen that the solution is near to but below $x = 3\pi/2(\sim 4.7)$. We therefore start the Newton-Raphson iteration with a value $x_0 = 4.5$.

The equation is rewritten as $\tan x - x = 0$. Let $f(x) = \tan x - x$ then $f'(x) = \sec^2 x - 1 = \tan^2 x$.

The Newton-Raphson iteration is $x_{n+1} = x_n - \dfrac{\tan x_n - x_n}{\tan^2 x_n}, \quad x_0 = 4.5$

so $$x_1 = 4.5 - \frac{\tan(4.5) - 4.5}{\tan^2 4.5} = 4.5 - \frac{0.137332}{21.504847} = 4.493614 \text{ to 7 sig.fig.}$$

Rounding to 4 sig.fig and iterating:

$$x_2 = 4.494 - \frac{\tan(4.494) - 4.494}{\tan^2 4.494} = 4.494 - \frac{0.004132}{20.229717} = 4.493410 \text{ to 7 sig.fig}$$

So we conclude that the value of x is 4.493 to 4 sig.fig. As $x = \mu L = (\sqrt{P/EI})L$ we find, after re-arrangement, that the smallest buckling load is given by $P = 20.19\dfrac{EI}{L^2}$.

FIGURE 2
FULLY WORKED SOLUTION PROVIDED FOR THE ENGINEERING FOCUSED EXAMPLE

Also included in each Section are student exercises (called 'tasks') which include space for students to write answers to the questions, and, where appropriate, guide them through problems in stages (see Figure 3). Additional exercises are included, usually at the ends of Sections, with solutions.

Task

An ellipse is described parametrically by the equations

$$x = 2\cos t \qquad y = \sin t \qquad 0 \le t \le 2\pi$$

Obtain an expression for the curvature κ and find where the curvature is a maximum or a minimum.

Your solution:

$\kappa =$

Solution

$$\text{Here } f(x) = (a^2 - x^2)^{\frac{1}{2}}$$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{-x}{(a^2 - x^2)^{\frac{1}{2}}} \quad \frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = \frac{-a^2}{(a^2 - x^2)^{\frac{3}{2}}}$$

$$\therefore \quad 1 + [f'(x)]^2 = 1 + \frac{x^2}{a^2 - x^2} = \frac{r^2}{a^2 - x^2}$$

$$\therefore \quad \kappa = \left| \frac{\dfrac{-a^2}{(a^2 - x^2)^{3/2}}}{\left[\dfrac{a^2}{a^2 - x^2}\right]^{3/2}} \right| = \frac{1}{a}$$

For a circle, the curvature is constant.

The value of κ (at any particular point on the curve, i.e. at a particular value of x) indicates how sharply the curve is turning. What this result states is that, for a circle, the curvature is inversely related to the radius. The bigger the radius, the smaller the curvature; precisely what, as we have argued above, we should expect.

FIGURE 3
A TYPICAL TASK FOR A STUDENT EMBEDDED IN A WORKBOOK

In some cases it is possible for the lecturer to select certain Sections from a Workbook and omit other Sections, reducing the reproduction costs and better tailoring the materials to the needs of a specific group.

On completion, HELM expects to have the following Workbooks and Guides:

- 45 Student Workbooks (listed in Appendix 1) written specifically with the typical engineering student in mind containing mathematical topics, worked examples, mathematical exercises and related engineering exercises.
- Engineering Case Studies ranging over many engineering disciplines primarily provided by an experienced engineering academics at Hull University.
- 1 Tutor Guide, providing commentary on each Workbook and associated CAL and CAA resources, relating success stories and challenges and encapsulating good practice derived from trialling in a variety of institutions with their individual contexts and cultures.
- 1 Student Guide providing advice and commentary on each Workbook and associated CAL and CAA resources.

In earlier trial versions the solutions to Tasks and Exercises were provided upsidedown. This has not proved popular with students and a decision has been made to provide them right-way-up for final trialling in 2004-05. The benefit of inversion is, of course, to lessen the chance of students inadvertantly seeing key features of the solution before engaging fully with the problem.

Workbooks vary in length from around 20 pages to 80 pages, with a median of around 40 pages. They are available in hardcopy and electronic formats.

One Workbook is devoted to Modelling Motion. Other engineering 'mini-case-studies' are included in various Workbooks where mathematical topics are presented. Two examples are given in Appendix 1.

A list of current Workbooks is given in Appendix 2.

HELM PROJECT CAL MATERIALS

The project currently has 72 CAL segments (mainly inherited from the original OLMP and are Authorware [6] based) which link to about half of the Workbooks. These enable web–based versions of the Workbooks to contain interactivity, audio and animations which generate student interest. Revision exercises with randomly generated questions are provided for the benefit of students working independently. These CAL segments have been found to be especially useful for supporting students of moderate mathematical ability, and for revision. They are also useful for illustrating lectures. However, following discussions with HELM triallists, the project team has decided to give low priority to the development of further CAL segments preferring to put its resources into developing the assessment regime described below.

THE HELM PROJECT ASSESSMENT REGIME

In formal educational environments assessment is normally an integral part of learning, and this is recognised by the HELM project. Students need encouragement and confirmation that progress is being made. The HELM assessment strategy is based on using Computer-Aided Asssessment (CAA) to encourage formative self-assessment, which many students neglect, to verify that the appropriate skills have been learned. The project's philosophy is that assessment should be at the core of any learning and teaching strategy. Loughborough University's own implementation of HELM makes extensive use of CAA to drive the students' learning through formative testing.

HELM provides an integrated web-delivered CAA regime for both self-testing and formal assessment. Students following the project's regime at Loughborough are typically tested four or five times each semester with questions delivered over the web. Currently there are around 5000 questions; many having detailed feedback. Students are encouraged to engage in their own learning by allowing them unlimited practice tests before taking a one-attempt summative test. As each summative test at Loughborough is worth about 6% of the module mark, students are motivated to keep up with their studies, thereby improving achievement and progression.

CAA is an essential part of the project and this raises potential difficulties over transferability, as each institution would need to support CAA delivery on completion of the project to gain the full benefit. The adoption of QM Perception at Loughborough University has allowed us to deliver tests to large numbers of students over the web since October 2000. Other institutions planning to use the HELM CAA regime would need to put in place an appropriate system in order to properly administer student test taking and to process the associated information through CAA.

It is envisaged that within the time-span of the project (2002-2005) most UK institutions will be in a position to be able to exploit the HELM Assessment regime one way or another.

Web-delivered CAA is a convenient method of delivery but an alternative implementation based on CDs has been developed. Currently all of the HELM tests (that is, all the questions and all the associated feedback) together with all the

Workbooks easily fit onto one CD. Students provided with such a CD can then do the required work and complete the tests on the CD, without an internet connection. This is easy to implement if only self-testing is required; formal testing is more challenging, requiring a network connection so that the students can submit their completed test results for processing. This way the test can be done off-line and students need not be live on the web for long periods; only the few seconds it takes to upload test results. This scheme has already been successfully incorporated in another Loughborough University based project, undertaken on behalf of the Royal Academy of Engineering, entitled BestMaths [7].

UNDERLYING STRUCTURE OF HELM CAA QUESTIONS

HELM CAA Questions have been designed to match particular mathematical concepts in support of the topics covered by the HELM Workbooks.

Within QM Perception a consistent naming convention has been adhered to which clearly identifies the location of the topic within the Workbook structure and describes the question so that its purpose is readily discernible.

The questions relevant to each mathematical (or statistical) concept have been structured into two sets, one nominally designated formative the other summative. Each set contains at least 10 questions cloned from a designated single master question, thereby ensuring a comparable level of difficulty is maintained, and justifying the random selection of questions from each set for test purposes. Several concepts may be selected and appropriate questions chosen randomly and presented within QM Perception as a customised test.

Question feedback is an option which may be enabled in both formative and summative type questions, but we strongly recommend that it always be used with formative questions as a motivational and pedagogical aid which drives student learning.

In many cases this feedback shows the specific worked solution or an example solution, while in simpler cases a generic solution may be presented. The importance of providing specific feedback for the benefit of the weaker learner is illustrated by the following student comments:

- I wish the practice tests gave better feedback, i.e. step by step showing with actual values rather than just partial workings out with algebra, especially for types of questions not found in the HELM Workbooks.
- I believe the feedbacks from the practice test can be improved by giving fully explanations to how to do it and along with working answer. From my past experiences, some of the questions I weren't able to answers (usually the harder ones) were lack of explanations and no working answer to the problem.

TYPES OF QUESTIONS USED FOR CAA

The simplest response required to a particular CAA question is the input of a numerical value, which may be either a whole number or a decimal.

An advantage of this Numeric entry type of question is that it is simple to construct and allows for the easy generation of clones with which to populate the relevant question library (bank). Also, when answers are requested to an accuracy of, say, 2 decimal places, it is not likely that learners will be able to guess the correct response.

However, there are disadvantages to the use of the Numeric entry type question that need to be addressed, particularly with regard to the accuracy required for the response, possible errors occurring while entering the answers (sticky keys, accidental or deliberate extra spaces, transcription errors, alternate symbolic conventions) and the issue arises as to exactly what is being tested.

It is a common occurrence for a learner to understand all of the mathematics related to a question but simply to fail to round their numeric answer correctly at the last stage where for example 2.136 required to 2 decimal places is entered as 2.13, or, more subtly, 2.1346 is firstly represented as 2.135 and then is entered as 2.14.

Initially we accepted precise answers only, believing it to be important that engineering undergraduates understand the need for precision. However, feedback from students indicates that this policy is a matter of some concern to them, particularly when taking summative tests, undermining the objective of driving their learning:

- With the CAA tests, being off the answers by 0.01 can result in your answer being incorrect, causing you to lose a lot of marks even though your method and approach are correct. I think it would be better to have a range of answers for questions that require you to round off your answer.
- The online CAA tests do not take account of an understanding of the subject matter, only an ability to produce an exact answer required. This does imply understanding, but making an error to a 100th does not imply misunderstanding.
- Like others - I have strong queries about the CAA. In one test I had all of the correct working, but got an answer which was 0.01 out due to rounding. I received no marks. I have done this in a few tests and am upset and concerned about my marks.

In an attempt to address this problem we now allow for responses within a tolerance where rounding is required, for example, ±0.01 for questions requiring 2 d.p. accuracy. We attempt to alert learners to this source of error by indicating in the feedback to them that, while their answer has been allowed, there is the possibility that they have made a rounding error.

In some circumstances a single Numeric response is inappropriate, for example in the factorising of a quadratic expression. In such circumstances the Multiple Choice type of question has been used. This approach has the advantage that it avoids the earlier Numeric input issue, but is of course susceptible to guesswork. A further requirement of this type of question is the construction of realistic, and not obviously wrong, distractors, preferably based on knowledge of typical errors and misconceptions. As a consequence, this type of question is more difficult to produce, and is especially challenging when many clones are needed.

OTHER NON-STANDARD HELM CAA QUESTIONS

We noted earlier the inherent advantages of Numeric input type question and also the limitations of single input with certain mathematical concepts. Looking at some examples: in the case of complex numbers, it is desirable to check both real and imaginary parts of the answer so marks can be given according to the accuracy of each component. Similarly, where there are two complex numbers involved, such as asking for the roots of a quadratic equation, it may be useful to mark individually the real and imaginary parts of each complex number and that means allowing for entry of four separate numbers as the answer. Finding coordinates of a point is another example where multi-input questions would be the best approach. Figure 4 shows an example of a HELM multi-input question.

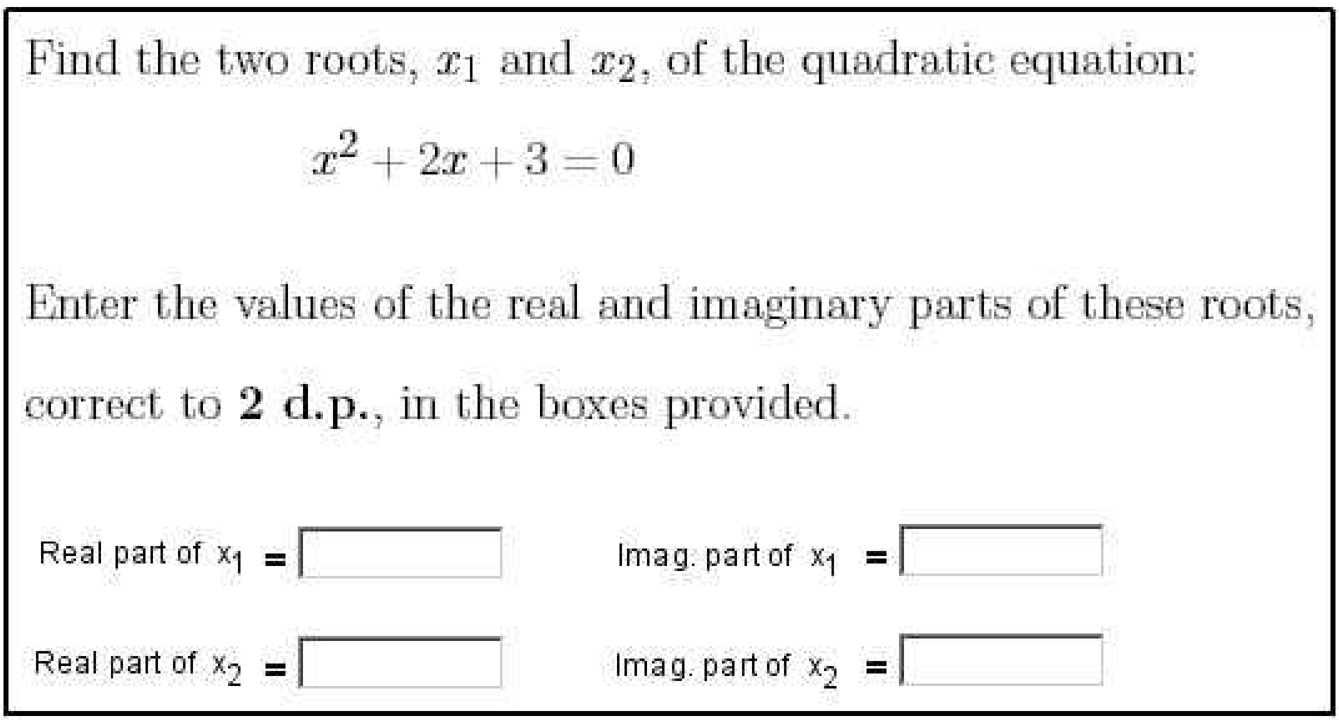


FIGURE 4
HELM CAA QUESTION REQUIRING FOUR NUMERIC INPUTS

Multi-input type questions can also be used in place of existing MCQs to eliminate the associated guesswork element. For example, if a quadratic equation is expected as the answer, instead of giving (say) five possible quadratic equations using an MCQ approach, three numeric inputs can be used to mark the coefficients of the terms of the quadratic equation. With this approach, the student cannot guess the correct quadratic equation. (There is a slight problem here in that strictly speaking the coefficients are not unique.)

With multi-input questions, the feedback given to the student can be designed to indicate which components of the answers were correct and marks can be allocated accordingly. However, setting conditions for computerised marking becomes difficult compared to single input numeric entry type. Unless the input areas are clearly labelled, students might input the answers in the boxes in any order, and automated marking conditions have to be set keeping this in mind. One good example emphasising the complexity of this type of question is when asking for the three roots of a cubic equation. As it is not possible to define which is the first root and so on, even if the input boxes are clearly labelled as root-1, root-2 and root-3, the student may designate any of the roots as root-1 depending on the way the question is tackled. This means, all given answers have to be marked collectively while checking for duplicates and allowing the student to enter the roots in any order.

In order to minimise students' frustration when they do not get marks when a rounding error is made, we introduced marks for numeric answers within a tolerance and this was discussed earlier. With a standard single numeric entry question, setting tolerances for an answer is straightforward. However, when two or more numeric answers are expected, setting tolerances becomes a complex task as each possible scenario has to be defined within the question marking algorithm. Setting conditions and arranging case specific feedback for automated marking becomes even more difficult, especially when tolerances to answers are allowed with multiple numeric input questions.

HELM MULTI-STAGE CAA QUESTIONS

A disadvantage inherent in single stage questions (where the final answer only is expected) is that a wrong response does not give credit for any correct work that might have been done by learners prior to submission of their answer. In some questions where several calculation processes may have been required before the final result is obtained, the loss of all credit seems unfair, and is again an issue that has been commented upon by students:

- The tests are not a representation of my understanding - just number plugging - you can be 99.99% correct but gain no marks. There should be a tolerance in marking, as no marks for methods can be achieved.
- The tests would be far better if they took into account working out.
- The computer tests should be … developed so that marks for workings can be given. Currently it is very easy to obtain low marks, despite having a good grasp of the subject.

In an attempt to address this situation some questions are now being written which are of a Multi-Stage format, whereby partial credit is given for a correct response at each of several stages within a question (see below). The learner, having perhaps submitted an incorrect answer at an intermediate stage, is subsequently presented with sufficient information at the commencement of the next stage to allow him or her to continue, thus giving the opportunity to gain partial credit within a more complex question.

EXAMPLE OF A HELM MULTI STAGE QUESTION

Objective: To determine the value of the second derivative of $y = x^2 + \sin x$ when $x = 1$.

A preamble gives any specific information on answering this type of question and then the whole question is presented.

> This is a multi-stage question.
>
> Credit will be given for each correctly completed stage.
> If you begin the question you must go on to completion.
> You may not return to a stage after submitting the answer.
> You may not return to the question at a later time.
> Click on the NEXT button to see the question.

> Determine the value of the second derivative of $y = x^2 + \sin x$ when $x = 1$.
>
> Click on the NEXT button to begin stage 1.

The first part of the question is presented in Stage 1.

STAGE 1	Determine the first derivative of $y = x^2 + \sin x$.
	A) $\frac{x}{2} + \cos x$
	B) $\frac{x^3}{3} + \cos x$
	C) $2x + \sin x$
	D) $2x - \cos x$
	E) None of the above
	Select one of the 5 options, then click SUBMIT.
	This stage is worth 2 mark(s)

The correct response in this example is E and after submitting the answer the student moves to Stage 2.

The correct solution for Stage 1 is revealed to the student who now has the task of determining the second derivative, in Stage 2.

STAGE 2	The correct answer to stage 1 was $2x + \cos x$.
	Now determine the second derivative of $y = x^2 + \sin x$.
	A) $2 + \sin x$
	B) $2 – \cos x$
	C) $2 – \sin x$
	D) $2 + \cos x$
	E) None of the above
	Select one of the 5 options, then click SUBMIT.
	This stage is worth 1 mark(s)

The correct response in this example is C and after submitting the answer the student moves to Stage 3.

The correct solution for Stage 2 is revealed to the student who now has the task of determining the value of the second derivative when x = 1 in Stage 3.

STAGE 3	The correct answer to stage 2 was $2 – \sin x$.
	Now determine the value of the second derivative of $y = x^2 + \sin x$ when x = 1.
	Enter your answer correct to **2 d.p.** in the box below, then click SUBMIT.
	Answer [ ]
	This stage is worth 1 mark(s)

This being the last stage, the question is now completed and the student moves on to the next question.

LOUGHBOROUGH'S IMPLEMENTATION OF THE ASSESSMENT REGIME

In a typical testing regime students are given a Workbook for a new mathematics topic in Week 1 (for self-study or lecture support). Week 3 is then a Test week (during which lectures and tutorials run as normal, but on a new Workbook). The Test week is organised as follows:

- From Thursday to the following Wednesday a practice test is available on the web. Students may access this test at any time within this period and, as it is web-delivered, anywhere there is an internet connection. It can be practised as often as the student desires. No record is kept by staff on student performance on these practice tests (although it could be – for research and development purposes, for example). Some students simply access the practice test to get information on question types and level of difficulty without attempting to answer any questions. Most will make a serious attempt at the practice test at least once; many up to five times. Many will work in small groups sorting out difficulties with the practice test. A good number seek help with the practice test from staff in our Mathematics Learning Support Centre [8]. Others will access the test, input spurious answers just in order to get the feedback or possibly to try to discover all the possible questions! We find 95% of students engage in some way. This as a valuable learning mechanism and it is clear that students now engage with the learning process at some level throughout the semester.
- On Thursday and Friday the actual coursework test is available. Again, students may access this test at anytime within this period and from anywhere. However, they are only allowed to take this test once.

Both tests have an identical form, selecting questions randomly from previously created question banks covering aspects of the topic just covered in lectures. If a student gets a question wrong on the trial test a single page of feedback is available. The feedback may be exemplary (addressing the solution of similar problems to the one presented) or specific (in which the solution to the given problem is detailed). The only feedback available on taking a formal coursework test is the overall score and an indication of which questions were answered correctly and which incorrectly.

Although there are many possible question types we generally use just two: numeric input (the majority) and multiple choice. Multiple response and matrix questions may be used in the future. Some multi-stage questions are being incorporated.

Following extensive feedback exercises we find this testing regime to be generally popular with both staff and students. Students particularly like the flexibility this method of assessment offers. They like the facility to practice tests and the possibility of doing tests when they are ready.

An occasional and valid concern raised by academics is the current practice at Loughborough of using the same question banks for both informal testing and formal testing. It is intended to have separate question banks in the future (and tell the students) to discourage a rote-learning approach. Another legitimate concern is allowing Loughborough students to undertake the formal tests unsupervised. There are great benefits to the students to allow them this freedom but at least some supervised tests would seem wise. This is very much up to the individual academic or institution to decide upon.

A test covering, say, 10 mathematical concepts, with a question randomly chosen from 10 questions available in each library, thus provides a determined student with the opportunity to attempt a possible 100 different questions, and to study question specific feedback prior to formal assessment.

Since students know that the summative questions are of a similar nature they are well motivated to undertake the formative tests and so gain confidence (and competence) prior to summative testing.

Analysis of student logs shows intensive activity during this practice test period. Feedback from students demonstrates how much this aspect of the assessment is appreciated:

- The practice tests are a really helpful tool to mastering the subject at hand. Many people would just revise just for the test and not learn a great deal, but by having practice tests it makes it a less formal way to prepare for the test but is also aiding revision for the module. Should definitely be kept.

DISSEMINATION OF THE HELM CAA REGIME

An engineering academic at another UK institution using HELM resources has commented:

- Students undertake two phased tests per semester, which are paper based and supervised. However, the HELM CAA system is used to provide practice tests to help the students prepare for the formal tests. These tests are available online and students need to login to access them. (Salford University)

For HEIs in England and Northern Ireland, HELM can provide question banks in QTI [9] XML format, for importing into their QM Perception installations or into other QTI compliant CAA software. Also, predefined tests on a stand-alone CD ROM can be provided.

From a staff perspective, CAA testing allows for monitoring of student understanding at intervals within the teaching period. Identification of misconceptions is thus possible and enables remedial action to be taken.

Within such a flexible testing regime, allowing students the opportunity to take unsupervised summative tests carries some risk. In line with the requirements of the code of practice for the use of information technology (IT) in the delivery of assessments (BS7988 [10]), we must be confident that our assessments are reliable and fair.

- Colleges, universities, schools and businesses are increasingly using computers to deliver exams and assessments in place of pen and paper. This standard (BS7988) will set guidelines to follow so that students know they will be treated fairly and so that organizations can trust and rely on the results of computer-delivered assessments.
(John Kleeman, Chair of British Standards Institute Committee).

The following student comments reveal that this may not be the case in spite of clear guidelines which are issued to students prior to taking the summative test:

- Many students also take the summative tests together, making it hugely unfair for students who follow the guidelines.
- I do also feel that many on my course are sitting this test together to aid their "learning" and results. I do not do this and I feel the marks I gain are my own, perhaps if I want higher marks I should start doing what is perceived as "normal" in the student body and effectively cheat by sitting the test with others' support. I wish to make you aware that a good deal of the students (if I was to rough a guess - maybe 60%) are gathering to take these tests - It is not fair on the rest of us who only have on mind working on these tests.

It would seem to be necessary to design into a formal testing system a mechanism for invigilation if this unfair practice is to be eliminated.

The HELM practice tests do not need supervision as their main purpose is to drive learning through continuous formative assessment, where collaboration, discussion and access to external resources are to be encouraged.

TRIALLING AND EVALUATION

The HELM learning resources are on trial at the five consortium members. In addition, we have made arrangements to trial materials in forty universities and colleges (see Appendix 3) Strictly speaking, this project is limited to HEIs in England and Northern Ireland but there has been considerable interest from Scotland, and some from Wales and the funding body (HEFCE) have agreed that institutions in those countries can be included in the trialling. Interest abroad has been shown but the project's intention is to wait until it nears the end of its final year (2004-05) before considering marketing outside the UK.

Trialling will determine whether these resources can be successfully used across the HE sector and in particular will enable us to assess the viability of the assessment regime elsewhere. So far, the evidence is promising.

ACKNOWLEDGEMENT

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APPENDIX 1

Examples of engineering mini-case-studies included in various Workbooks

ENGINEERING MINI-CASE STUDY ON POLYNOMIALS

Introduction

Many aspects of physics and engineering involve inverse square law dependence. For example gravitational forces and electrostatic forces vary with the inverse square of distance from the mass or charge. This mini-case study concerns the dependence of sound intensity (I W/m^2) on distance (r m) from a source. For a single source of sound power W watts, this is expressed by

$$I = \frac{W}{4\pi r^2}.$$

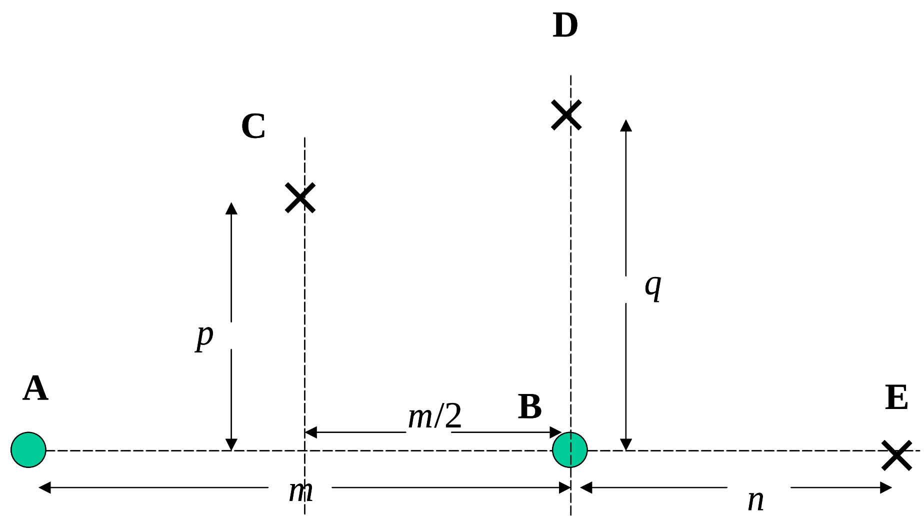
For two independent sources A and B (i.e. incoherent sources with no phase relationship) then the combined sound intensity (I_C W/m^2) at a specific point is given by the sum of the intensities due to each source at that point. So

$$I_C = I_A + I_B = \frac{W_A}{4\pi r_A{}^2} + \frac{W_B}{4\pi r_B{}^2},$$

where W_A and W_B are the respective sound powers of the sources; r_A and r_B are the respective distances from the point of interest. The independence or incoherence would apply to two separate items of industrial equipment for example but would not apply in the case of two loudspeakers connected to the same amplifier.

Engineering Problem Posed

With reference to the situation shown in the Figure below, (i) find the sound intensities at given points C and D at distances p and q from the line joining two incoherent point sources at A and B with sound powers 1.9 W and 4.1 W respectively separated by 6m and (ii) find the locations of C, D and E that correspond to sound intensities of 0.02, 0.06 and 0.015 W/m^2 respectively.



Engineering Problem Expressed Mathematically

Using the symbols defined in Figure 1,

(a) Find an expression for the sound intensities at point C due to the independent sources A and B.

(b) Find the expression for p such that the intensity at point C is a given value I_C.

(c) If W_A = 1.9 W, W_B = 4.1 W and m = 6m calculate the distance p at which the 0.02 W/m^2?

(d) Find an expression for the intensity at point D.

(e) Find the value for q such that the intensity at D is 0.06 W/m^2 and the other values are as in part (c).

(f) Show that a general expression for the distance n at which the intensity at point E is I_E is given by the appropriate roots of

$$4\pi I_E n^4 + 8\pi I_E mn^3 + \left[4\pi I_E n^2 - \left(W_A + W_B\right)\right]n^2 - 2mW_B n - m^2 W_B = 0$$

(g) By plotting this function for I_E = 0.015 W/m^2, m = 6m, W_A = 1.9 W, W_B = 4.1W, find the corresponding values for n.

Mathematical Analysis

(a) The combined sound intensity I_C W/m^2 is given by the sum of the intensities due to each source at C. Because of symmetry of the position of C with respect to A and B, write AC = BC = r, then

$$I_C = I_A + I_B = \frac{W_A}{4\pi r_A^2} + \frac{W_B}{4\pi r_B^2} = \frac{W_A + W_B}{4\pi r^2}$$

Using Pythagoras,

$$r^2 = \left(\frac{m}{2}\right)^2 + p^2$$

Hence

$$I_C == \frac{W_A + W_B}{4\pi\left(\left(\frac{m}{2}\right)^2 + p^2\right)} = \frac{W_A + W_B}{4\pi\left(m^2 + 4p^2\right)}$$

(b) Making p the subject of the last equation,

$$p = \pm\frac{1}{2}\sqrt{\left(\frac{W_A + W_B}{\pi I_C}\right) - m^2}$$

(c) Using the given values, $p \approx 3.86\text{m}$. The negative value is a consequence of the sound field symmetry about the line joining the two sources.

(d) Using Pythagoras again, the distance from A to D is given by $\sqrt{q^2 + m^2}$. So

$$I_D = I_A + I_B = \frac{W_A}{4\pi r_A^{\;2}} + \frac{W_B}{4\pi r_B^{\;2}} = \frac{W_A}{4\pi\left(q^2 + m^2\right)} + \frac{W_B}{4\pi q^2}$$

(e) Making q the subject of this equation produces a quartic equation,

$$4\pi I_D q^4 + \left[4\pi m^2 I_D - \left(W_A + W_B\right)\right]q^2 - W_B m^2 = 0$$

Since the quartic contains only even powers, it can be regarded as a quadratic in q^2 and this can be solved by the standard formula. Hence

$$q^2 = \frac{-\left[4\pi m^2 I_D - \left(W_A + W_B\right)\right] \pm \sqrt{\left[4\pi m^2 I_D - \left(W_A + W_B\right)\right]^2 + 16\pi I_D W_B m^2}}{8\pi I_D}$$

Using the given values,

$$q^2 \approx \frac{-21.14 \pm 29.87}{1.51}$$

Since q must be real, the negative result can be ignored. Hence $q \cong 2.40$ m.
(f) Using the same procedure as in (d) and (e),

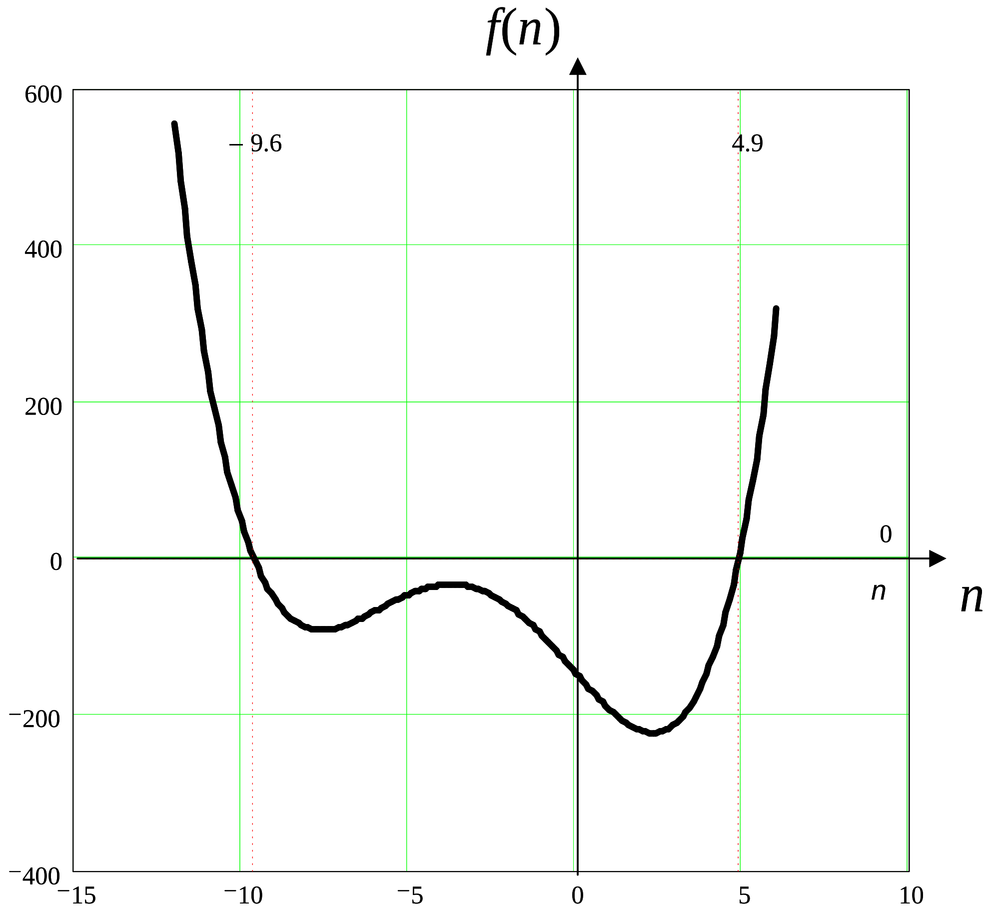
$$I_E = I_A + I_B = \frac{W_A}{4\pi r_A^{\;2}} + \frac{W_B}{4\pi r_B^{\;2}} = \frac{W_A}{4\pi\left(m+n\right)^2} + \frac{W_B}{4\pi n^2}$$

$$4\pi I_D n^2 \left(m+n\right)^2 I_E = W_A n^2 + \left(m+n\right)^2 W_B = 0$$

The required expression follows from collecting together coefficients of particular powers of n.
(g) Substitution of the given values produces

$$0.1885n^4 + 2.2619n^3 + 0.7858n^2 - 49.2n - 147.6 = 0.$$

The plot of the quartic below shows that there are two roots of interest. One is at $n \cong 4.9$ m and the other is at $n \cong -9.6$ m.



Interpretation

The result for part (g) implies that there are two locations E along the line joining the two sources where the intensity will have the given value. One position is about 3.6m to the left of source A.

Engineering Mini Case Study on Bending Moments (Workbooks 14 & 15)

Introduction

Two quantities are of interest to the engineer when designing structures in the form of beams or trusses. These are

- The Shear Force, denoted by S and measured in N
- The Bending moment, denoted by M and measured in Nm

The shear force is the negative of the derivative (with respect to position) of the bending moment and the load is the derivative of the shear force. Consider a beam which is supporting a uniform load per unit length of w measured in N/m. This load may represent the self-weight of the beam or may be an external load. The quantities M, S and w are related by

$$\frac{dM}{dz} = S \qquad (1)$$

and

$$\frac{dS}{dz} = -w \qquad (2)$$

where z measures the position along the beam.

Equation (1) implies that the bending moment is the integral of the shear force. Equation (2) implies that the shear force is the negative of the integral (with respect to position) of the load. The negative sign in equation (2) reflects the fact that the usual convention is to consider loads in the downward direction to be positive whereas a shear force is considered to be positive in the upward direction. If one of the quantities is known, the others can be calculated from equations (1) and (2).

As a first example consider the cantilever shown in Figure 1. This consists of a uniform beam of length L, supporting its own weight i.e. w_0 per unit length (= constant). One end ($z = 0$) of the beam is rigidly fixed and the other end ($z = L$) is free to move. At the free end the shear force and bending moment must be zero. This may be regarded as a *boundary condition.*

Equations (1) and (2) may be used to find the shear force S and the bending moment M as functions of z.

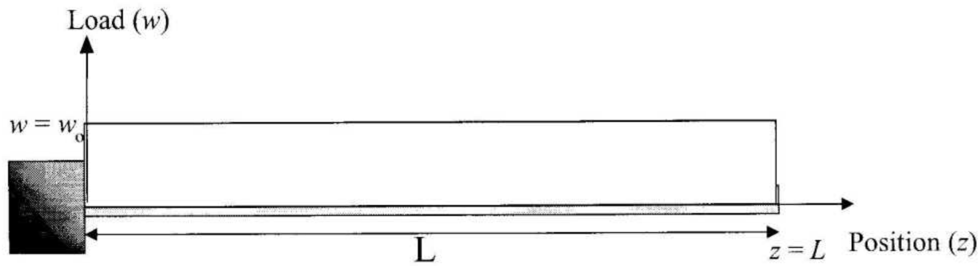


Figure 1 A simple cantilever with uniformly distributed load

As w is a constant, equation (2) gives $S = -\int w dz = -\int w_0 dz = -w_0 z + C$. At the free end ($z = L$); the shear force $S = 0$ so $C = w_0 L$ giving $S = w_0(L - z)$.

This expression can be substituted into equation (1) to give

$$M = \int S dz = \int w_0 (z - L) dz = \int (w_0 z - w_0 L) dz = \frac{1}{2} w_0 z^2 - w_0 L z + K .$$

Using M = 0 at the free end ($z = L$), K is given by $K = \frac{{w_0}^2 L^2}{2}$.

Thus

$$M = w_0 L z + \frac{w_0}{2}\left(L^2 - z^2\right).$$

The diagrams in Figure 2a and 2b show the shear force S and the bending moment M respectively as functions of position z.

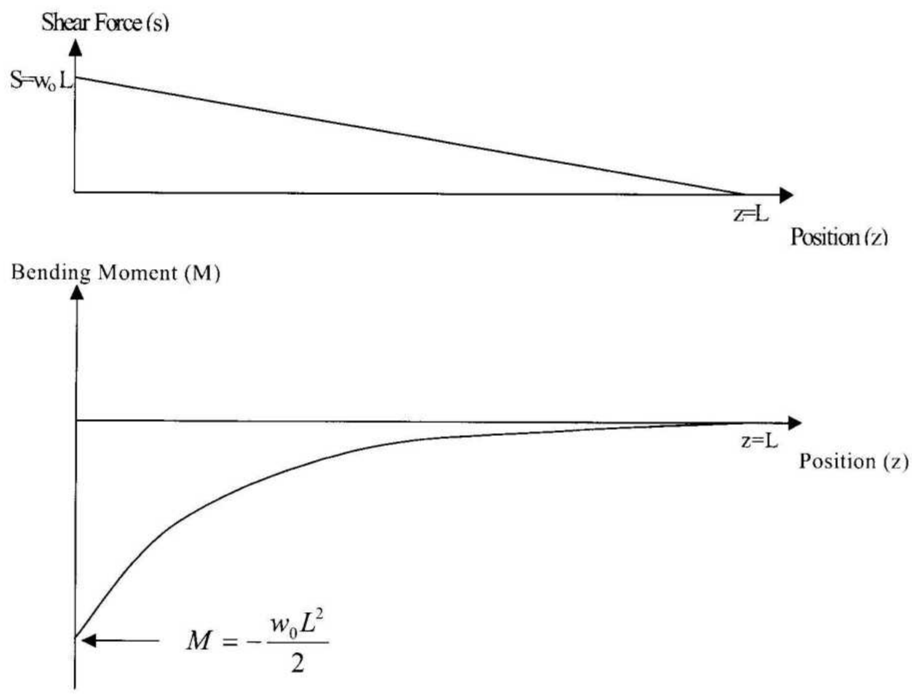


Figure 2 Shear Force and Bending Moment as functions of position along a simple cantilever with uniformly distributed load.

As would be expected intuitively the shear force and Bending Moment have their maximum values at the fixed end. The shear force varies linearly with position, whereas the Bending moment has a quadratic dependence on position.

Suppose now that a load W is concentrated at a point at the free end of the cantilever rather than distributed. The point load can be approximated by a load per unit length of W/δ spread over a small length δ at the free end. As δ approaches zero, the situation tends to the required one.

Over the small portion δ at the free end of the beam, the integral form of Equation (2) gives $S = -\int \frac{W}{\delta} dz = -\frac{W}{\delta} z + C_1$. As before, at $z = L$, $S = 0$, so $0 = -\frac{W}{\delta} L + C_1$ or $C_1 = -\frac{W}{\delta} L$. So over the small hypothetical small portion δ at the free end of the beam, $S = \frac{W}{\delta}(L - z)$. In particular at $z = L - \delta$ which represents the beginning of this fictitious portion, $S = \frac{W}{\delta}\left(L - (L - \delta)\right) = W$.

Along the main portion ($0 \le z \le L-\delta$) of the beam, $S = -\int 0 dz = C_2$.

Since the shear force must be continuous at $z = L-\delta$, where $S = W$, $C_2 = W$.

In summary,

$$S = \frac{W}{\delta}(L-z) \qquad L-\delta \le z \le L$$

$$W \qquad 0 \le z \le L-\delta$$

For $L-\delta \le z \le L$, the appropriate expression for S may be substituted in the integral form of equation (2) to give

$$M = \int \frac{W}{\delta}(L-z)dz = \frac{W}{\delta}\left(Lz - \frac{z^2}{2}\right) + C_3 .$$

Since $M = 0$ when $z = L$, $0 = \frac{W}{\delta}\left(L^2 - \frac{L^2}{2}\right) + C_3$, so $C_3 = -\frac{WL^2}{2\delta}$ and

$$M = \frac{W}{\delta}\left(Lz - \frac{z^2}{2}\right) - \frac{WL^2}{2\delta} = -\frac{W}{2\delta}(L-z)^2 \qquad L-\delta \le z \le L .$$

For the main part of the beam ($0 \le z \le L-\delta$) along which $S = W$, the integral form of equation (2) gives

$M = \int W dz = Wz + C_4$.

For continuity at $z = L-\delta$ where $M = -\frac{W}{2\delta}(L-(L-\delta))^2 = -\frac{W\delta}{2}$,

$-\frac{W\delta}{2} = W(L-\delta) + C_4$ or $C_4 = \frac{W\delta}{2} - WL$.

Hence

$$M = W\left(z - L + \frac{\delta}{2}\right) \qquad 0 \le z \le L-\delta .$$

If $\delta \to 0$, then at any point along the whole cantilever,

$$S = W$$

and

$$M = W(z-L).$$

The shear force result is independent of position and is consistent with simply considering the equilibrium of the beam. The bending moment prediction is varies linearly with position and is consistent at any position with the results of 'taking moments' about that position. Although there are simpler ways of obtaining these results for the point load, the integration procedure that has been introduced can be used for more complicated problems (see engineering problem (b)).

Engineering Problems posed

(a) A beam of length L is constructed such that the load is given by $w = w_0\left(1 + \sin\frac{\pi z}{L}\right)$ i.e. it is heavier in the middle than at the ends.

Find expressions for the shear force and bending moment as functions of position z and sketch the results.

(b) A light beam of length L supports a point load W at the free end and a continuous load of w_0 (measured in N/m) spread over the half of the beam nearest the fixed end ($0 < z < L/2$).

Find expressions for the shear force and bending moments as functions of position z and sketch the results.

Engineering Problems Expressed Mathematically

(a) Use equations (1) and (2) and $w = w_0\left(1 + \sin\dfrac{\pi z}{L}\right)$ with the conditions $S = 0$ and $M = 0$ at $z = L$ to find the shear force and bending moments as functions of position z.

(b) Use equations (1) and (2), $w = w_0$ $0 < z < L/2$, point load W at $z = L$, the conditions $S = 0$ and $M = 0$ at $z = L$ and continuity of S and M at $z = L/2$, to find the shear force and bending moments as functions of position z.

Mathematical Analysis

(a) Equation (2) gives $S = -\int w_0\left(1 + \sin\dfrac{\pi z}{L}\right)dz = -w_0\left[z - \dfrac{L}{\pi}\cos\dfrac{\pi z}{L}\right] + C$.

At the free end ($z = L$), $S = 0$, so $0 = -w_0\left[L + \dfrac{L}{\pi}\right] + C$ and

$$S = -w_0\left[z - \frac{L}{\pi}\cos\frac{\pi z}{L}\right] + w_0\left[L + \frac{L}{\pi}\right].$$

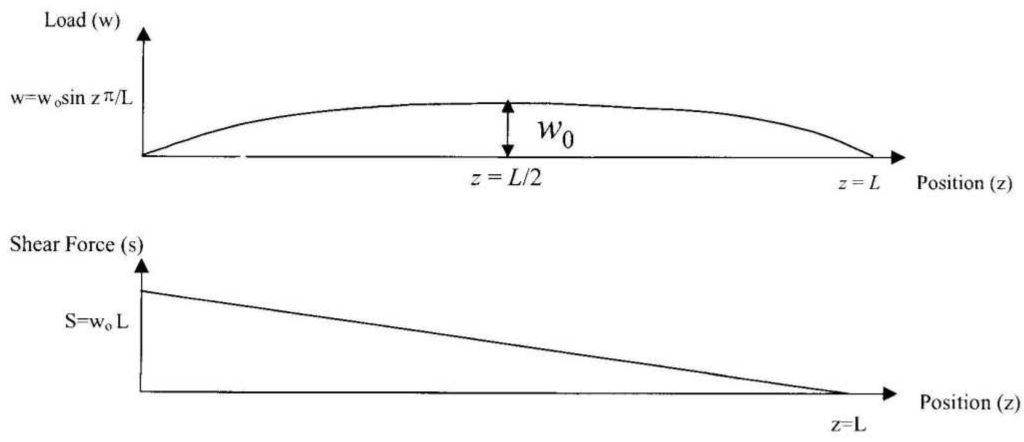
This can be substituted in equation (1) to give

$$M = \int\left\{- w_0\left[z - \frac{L}{\pi}\cos\frac{\pi z}{L}\right] + w_0\left[L + \frac{L}{\pi}\right]\right\}dz = - w_0\left[\frac{z^2}{2} + \frac{L^2}{\pi^2}\sin\frac{\pi z}{L}\right] + w_0 Lz\left(1 + \frac{1}{\pi}\right) + K$$

At the free end ($z = L$), $M = 0$, so $0 = -w_0\left[\dfrac{L^2}{2}\right] + w_0 L^2\left(1 + \dfrac{1}{\pi}\right) + K$ and

$$M = -w_0\left[\frac{z^2}{2} + \frac{L^2}{\pi^2}\sin\frac{\pi z}{L}\right] + w_0 Lz\left(1 + \frac{1}{\pi}\right) - w_0 L^2\left(\frac{1}{2} + \frac{1}{\pi}\right).$$

These results for Shear Force and Bending Moment are sketched in Figure 6 (as is the load).



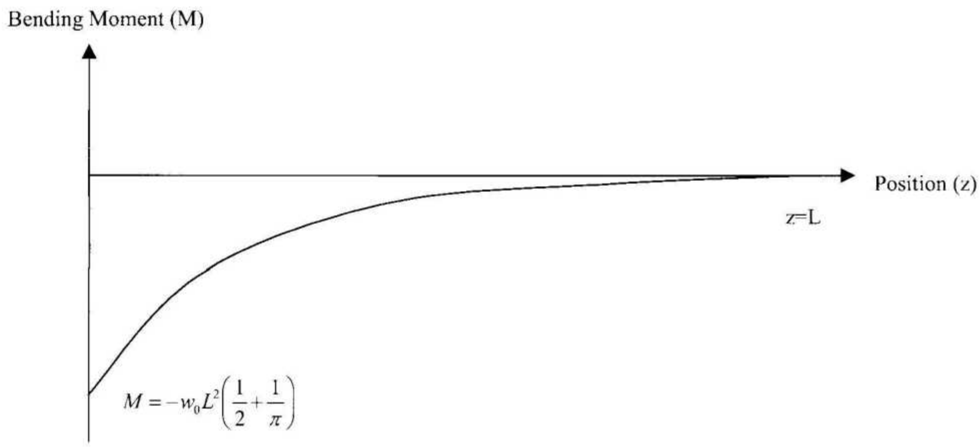


Figure 6 The load, shear force and bending moment as a function of position for problem (a)

(b) As in the simpler example considered in the Introduction, at the free end, the load of W can be considered as a load of W/δ spread over the part of the beam $L-\delta \le z \le L$.

Over the small portion δ at the free end of the beam, the integral form of Equation (2) gives $S = -\int \frac{W}{\delta}dz = -\frac{W}{\delta}z + C_1$. At $z = L$, $S = 0$, so $0 = -\frac{W}{\delta}L + C_1$ or $C_1 = -\frac{W}{\delta}L$.

So over the small hypothetical small portion δ at the free end of the beam, $S = \frac{W}{\delta}(L-z)$. In particular at $z = L - \delta$ which represents the beginning of this fictitious portion, $S = \frac{W}{\delta}(L-(L-\delta)) = W$. This value of S must match with that on the adjacent part of the beam.

For the part of the beam $L/2 \le z \le L-\delta$, the load is zero, so equation (2) gives $S = -\int 0 dz = C_2$: As this must equal W at $z = L - \delta$, $C_2 = W$ and so $S = W$.

For $0 < z < L/2$, the integral form of equation (2) states that $S = -\int w_0 dz = -w_0 z + C_3$. As this must join on continuously to the part of the beam at $L/2 \le z \le L-\delta$, S must equal W at $z = L/2$. Hence, $W = -w_0\frac{L}{2} + C_4$ and $S = -w_0 z + W + \frac{w_0 L}{2}$.

So

$$S = \begin{cases} -w_0 z + W + \frac{w_0 L}{2} & 0 < z < L/2 \\ W & L/2 \le z \le L-\delta \\ \frac{W}{\delta}(L-z) & L-\delta \le z \le L \end{cases}$$

For $L-\delta \le z \le L$, the integral form of equation (1) gives

$$M = \int \frac{W}{\delta}(L-z)dz = \frac{W}{\delta}\left(Lz - \frac{z^2}{2}\right) + C_5$$

Since $M = 0$ when $z = L$, $0 = \frac{W}{\delta}\left(L^2 - \frac{L^2}{2}\right) + C_5$, so $C_5 = -\frac{WL^2}{2\delta}$ and

$$M = \frac{W}{\delta}\left(Lz - \frac{z^2}{2}\right) - \frac{WL^2}{2\delta} = -\frac{W}{2\delta}(L-z)^2 \qquad L-\delta \le z \le L\,.$$

At $z = L - \delta$, $M = -\frac{W}{2\delta}\left(L-(L-\delta)\right)^2 = -\frac{W\delta}{2}$, and this value of M must join smoothly onto the part of the beam at $L/2 \le z \le L-\delta$.

In the region $L/2 \le z \le L-\delta$, the integral form of equation (1) gives

$M = \int W dz = Wz + C_6$.

For continuity at $z = L - \delta$ where $M = -\frac{W\delta}{2}$, $-\frac{W\delta}{2} = W(L-\delta) + C_6$ or

$C_6 = \frac{W\delta}{2} - WL$.

Hence

$$M = W\left(z - L + \frac{\delta}{2}\right) \qquad L/2 \le z \le L-\delta\,.$$

At $z = L/2$, $M = W\left(\frac{L}{2} - L + \frac{\delta}{2}\right) = \frac{W}{2}(\delta - L)$ and this must be continuous with the corresponding value for $0 < z < L/2$.

For $0 < z < L/2$, $M = \int\left[-w_0 z + W + \frac{w_0 L}{2}\right]dz = -\frac{w_0 z^2}{2} + \left(W + \frac{w_0 L}{2}\right)z + C_7$.

At $z = L/2$, $M = \frac{W}{2}(\delta - L)$, so $\frac{W}{2}(\delta - L) = -\frac{w_0}{2}\left(\frac{L}{2}\right)^2 + \left(W + \frac{w_0 L}{2}\right)\frac{L}{2} + C_7$. Hence

$C_7 = W\frac{\delta}{2} - \frac{w_0 L^2}{8} - WL$ and $M = -\frac{w_0 z^2}{2} + \left(W + \frac{w_0 L}{2}\right)z + W\frac{\delta}{2} - \frac{w_0 L^2}{8} - WL$.

So

$$M = -\frac{w_0 z^2}{2} + \left(W + \frac{w_0 L}{2}\right)z + W\frac{\delta}{2} - \frac{w_0 L^2}{8} - WL \qquad 0 < z < L/2$$

$$W\left[z + \frac{\delta}{2} - L\right] \qquad L/2 < z < L-\delta$$

$$-\frac{W}{2\delta}(L-z)^2 \qquad L-\delta < z < L$$

If $\delta \to 0$, then

$$S = -w_0 z + W + \frac{w_0 L}{2} \qquad 0 < z < L/2$$

$$W \qquad L/2 \le z \le L$$

and

$$M = -\frac{w_0 z^2}{2} + \left(W + \frac{w_0 L}{2}\right)z + -\frac{w_0 L^2}{8} - WL \qquad 0 \le z \le L/2$$

$$W[z - L] \qquad L/2 \le z \le L$$

The corresponding sketches are in Figure 7.

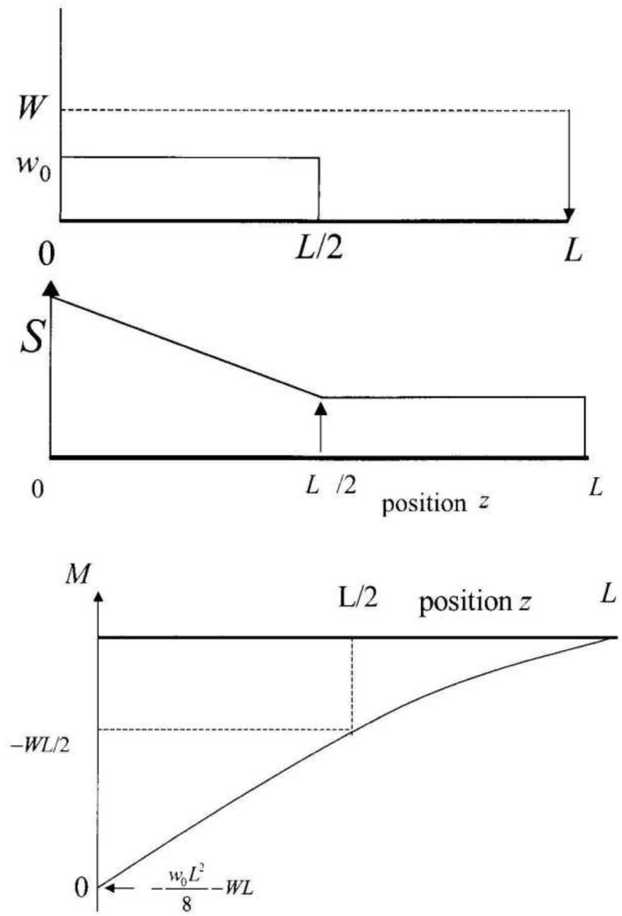


Figure 7 Load, Shear Force and Bending Moment for problem (b)

Interpretation

The shear force result for problem (a), i.e. the sinusoidal variation of load, is the same as for a uniformly distributed load of w_0 per unit length. However the bending moment result for problem (a) corresponds to a larger bending moment at the fixed end than for the uniformly distributed load.

For the half of the beam nearest the fixed end, the shear force result for problem (b), is the sum of the separate results for the uniformly distributed load and the point load. The Bending moment result for problem (b) at the fixed end is the sum of the contributions predicted for each load component. For the half of the beam nearest the free end, the bending moment result is the same as in the case of a single point load W at z = L.

APPENDIX 2

HELM Workbooks – as at 1st August 2004

No.	Title
1	Basic Algebra
2	Basic Functions
3	Equations, Inequalities and Partial Fractions
4	Trigonometry
5	Functions and Modelling
6	Exponential and Logarithmic Functions
7	Matrices
8	Matrix Solution of Equations
9	Vectors
10	Complex Numbers
11	Differentiation
12	Applications of Differentiation
13	Integration
14	Applications of Integration 1
15	Applications of Integration 2
16	Sequences and Series
17	Conics and Polar Coordinates
18	Functions of Several Variables
19	Differential Equations
20	Laplace Transforms
21	Z Transforms
22	Eigenvalues and Eigenvectors
23	Fourier Series
24	Fourier Transforms
25	Partial Differential Equations
26	Functions of a Complex Variable
27	Multiple Integration
28	Vector Calculus 1
29	Vector Calculus 2
30	Introduction to Numerical Methods
31	Numerical Methods of Approximation
32	Numerical Solution of Initial Value Problems
33	Numerical Solution of Boundary Value Problems
34	To be decided
35	Sets and Probability
36	Descriptive Statistics
37	Discrete Probability Distributions
38	Continuous Probability Distributions
39	The Normal Distribution
40	Sampling Distributions and Estimation
41	Hypothesis Testing
42	Goodness of Fit Tests and Contingency Tables
43	Correlation and Regression
44	ANOVA
45	Nonparametric Methods
46	Quality Control and Reliability
47	Case Studies 1 - Modelling Motion
48	To be decided
49	To be decided
50	Tutor's Guide
0	Student's Guide

APPENDIX 3

List of UK Higher Education Institutions participating in trials

Aston, Birmingham, Bournemouth, Bournemouth & Poole, City, Derby, Glamorgan, Glasgow, Glasgow Caledonian, Glenrothes, Harper Adams, Hertfordshire, Kingston, Lancaster, Leeds, Leeds Metropolitan, London Metropolitan, Moray, Leicester, Liverpool, Nottingham, Nottingham Trent, North Devon, Newcastle, Northumbria, Oxford Brookes, Portsmouth, Plymouth, Queen's Belfast, Robert Gordon, Salford, Southampton, Surrey, Ulster, UWIC and Westminster Kingsway.