Making maths meaningful – efforts to enhance the learning experiences of engineering students

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Abstract

Students' mathematical difficulties are a well-documented national issue. In this paper, the author considers his experiences of teaching mathematics over ten years to diverse groups of engineering undergraduates and aims to identify those aspects of teaching which contribute to an enhanced learning experience.

A first year mathematics module, in existence for ten years, has been provided to large classes of aerospace, chemical, civil and mechanical engineering students (typical class size is 250). In recent years, a large majority of the students have achieved at least grade B in A-level mathematics but the class usually includes a small group of students with much lower qualifications in maths. For example, in 2010/11 about 51% of the class possessed A-level maths grade A* or A, 35% had A-level maths grade B, while about 7% did not have an A-level maths qualification (or equivalent). Feedback from students indicates their preference to write notes during lectures rather than receive handouts – not only does this improve concentration, it also teaches the good practice of setting out a solution sequentially. Including background theory and deriving results, rather than being presented with formulas to learn, was also a key preference of students. Clear presentation of material, having plenty of good examples, and demonstrating application of the maths are also important for learning. Students appreciate mid-term class tests, understanding that they promote a more structured approach to learning. The mean pass rate for the course is 89% while the mean exam average mark is 65%.

The second year aerospace engineering students enrolled on the MEng programme (about 20 students each year) take a mathematics and computing module which involves more applied topics such as linear algebra, vector calculus and Laplace transforms. The small class size has allowed a more interactive approach and a mixture of formal teaching and student practice within the session. Student feedback suggests they enjoy the involving style of teaching. The computing section, based on Matlab, aims to demonstrate application of the maths and show the usefulness of computing tools in aerospace engineering. Students have found the Matlab section to be challenging but could identify gains from the course, in terms of both subject-specific and transferable skills.

In general, student feedback indicates that an enthusiastic and personal approach, establishing a good rapport with students, is significant. This requires attending tutorial classes, talking to students, encouraging them, even trying to learn their names. It has also been interesting to note that students make a distinction between a lecturer and a teacher.

1. Introduction

A decline, nationally, in students' mathematical skills has been extensively described [1]. This problem is believed to have worsened since the early 1990s. Students were found to be able to achieve good A-level grades in mathematics but be inadequately prepared for the mathematics involved in engineering degrees. In particular, a deficiency in basic mathematical skills, for example, numerical and algebraic manipulation and simplification, has been observed.

Cox has proposed eleven principles for teaching mathematics [2]. The teacher should be aware of the students' previous levels of achievement in the subject. There must be clear communication, both in lectures and regarding what is expected of students. He believes the maths should be presented in a well-explained, exploratory way rather than as bite-sized products. Students need to be engaged in practice and enthused, for example by good positive feedback. A professional and caring approach towards students is also advocated.

Liston and O'Donoghue [3] report the results of interviews with first year students on various engineering, technology and science degree programmes. Students often attributed their past experiences of maths to their teachers and stressed the importance of seeing how the maths could be applied in their careers. Holton [4] has hinted at potential benefits from incorporating problem-solving classes into the course and through peer tutoring but it is not clear whether these approaches can be successfully adapted to large classes. The potential for confusion when engineering students are taught by staff from a mathematics department has been described [5].

The author has ten years of experience teaching mathematics to diverse groups of engineering students. This paper focuses on a first year (Stage 1) module provided to large classes of aerospace, chemical, civil and mechanical engineering students (typical class size is 250) and a more specialised second year (Stage 2) module for aerospace students (class size is 20). The aims and teaching methodologies for each module are described. The learning experiences of the students are investigated through exam results data, student questionnaires and focus group interviews [6]. The paper aims to identify those aspects of teaching which contribute to an enhanced learning experience.

2. Further Mathematics 1

This second semester course, along with a first semester course (Engineering Mathematics 1), represents the mathematics teaching (20 CATS points in total) for first year aerospace and mechanical engineering students. Having been aware of student difficulties with mathematics – previous Stage 1 exam results were disappointing and changes to A-level mathematics were occurring – the Stage 1 mathematics courses were redesigned for the 2001/02 academic year. A new syllabus was developed; some material previously taught at Stage 1 was moved into Stage 2. The Further Mathematics 1 syllabus has remained unchanged in the last ten years although some adjustments have been made to the Engineering Mathematics 1 course.

In first year engineering mathematics, it is aimed to provide students with a good grounding in a range of fundamental topics relevant to engineering (logarithms, polynomial equations, trigonometry, complex numbers, differentiation and integration, differential equations, matrices, vectors and statistics) and to demonstrate some application to real-world situations. It is also desired that students' confidence in their mathematical ability should be enhanced.

Thus, an in-depth knowledge at Stage 1 is not the predominant goal; rather, it is desired that students be competent in the basics (eg, applying methods for solving differential equations, matrix operations, calculating vector products) in a range of relevant topics.

Further Mathematics 1 is provided for aerospace, chemical, civil and mechanical engineering students (typical class size is 250). Most students enter university directly from school. In recent years, a large majority of the students have achieved at least grade B in A-level mathematics but the class usually includes a small group of students with much lower qualifications in maths. For example, in 2010/11 about 51% of the class possessed A-level maths grade A* or A, 35% had A-level maths grade B, while about 7% did not have an A-level maths qualification (or equivalent) (Figure 1). This shows a recent improvement in entry qualifications. In 2007/08 about 48% of the class possessed A-level maths grade A, 25% had A-level maths grade B, while about 10% did not have an A-level maths qualification (or equivalent). The diverse nature of the class is emphasised by a few students with A-level further maths (about 5% of the total in 2010/11).

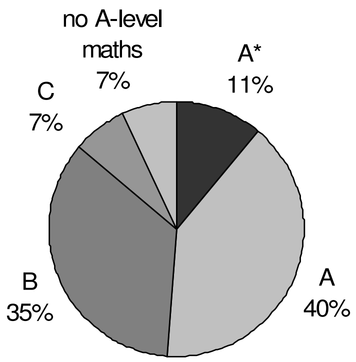

Figure 1: A-level maths grades for the students taking Further Mathematics 1 in 2010/11.

Teaching takes place over 12 weeks with a 2-hour lecture and 1-hour tutorial / exercise class per week. Students attend lectures as a single large group but are divided into smaller groups (of 30 – 40 students) for exercise classes.

2.1 Student Learning Experience

Exam results and pass rates have been pleasing and relatively consistent over the last nine years (Figure 2). The exam average mark has ranged from 59.9 – 68.2 % (mean 64.5%), while the pass rate has ranged from 82.7 – 93.8 % (mean 88.8%). One external examiner commented that the "average looks good, and there would appear to be relatively few failures, given the UK-wide picture of mathematics for engineers".

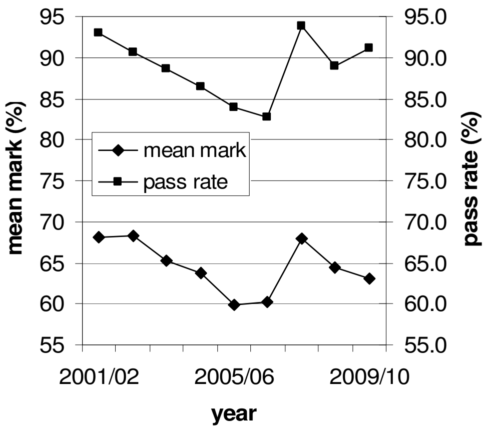

Figure 2: Mean mark and pass rate in Further Mathematics 1.

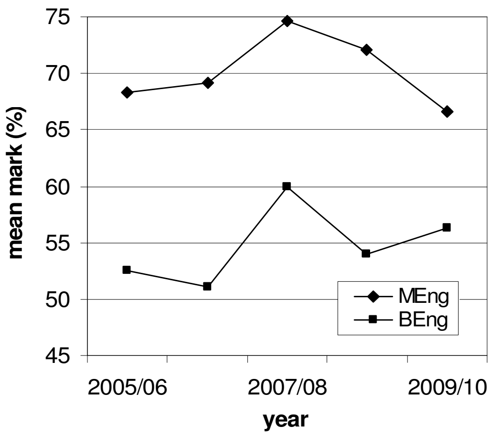

Figure 3: Mean mark in Further Mathematics 1 for BEng and MEng students.

A highly significant difference (P < 0.001%) between BEng and MEng performance (mean marks) is normal with the respective means typically differing by 15% (Figure 3). This is not surprising given the higher entry requirements for the MEng degrees.

Students are asked to rate various aspects of the course (for example, module aims clearly stated, module content presented clearly) on a scale of 1 (strongly disagree) to 5 (strongly agree). The questionnaires are distributed in the last or penultimate lecture and the response rate is typically 65%. There has been a high level of student satisfaction across all areas. In general, differences in the responses to each question from BEng and MEng students were not significant (at the 5% level).

On the same questionnaires, students had freedom to comment on "the most satisfactory aspects of this module". Typically, two-thirds of questionnaires will contain comments and the responses over the last four years have been collated into categories (Figure 4). Three categories stand out above all others. In the following discussion, examples of student comments associated with the various categories are provided and the focus group interviews are used to consider why the various aspects of the module may be effective. Interviews were conducted in weeks 10 and 11 of the semester (in April 2011) with a selection of students from both the current and previous year's classes.

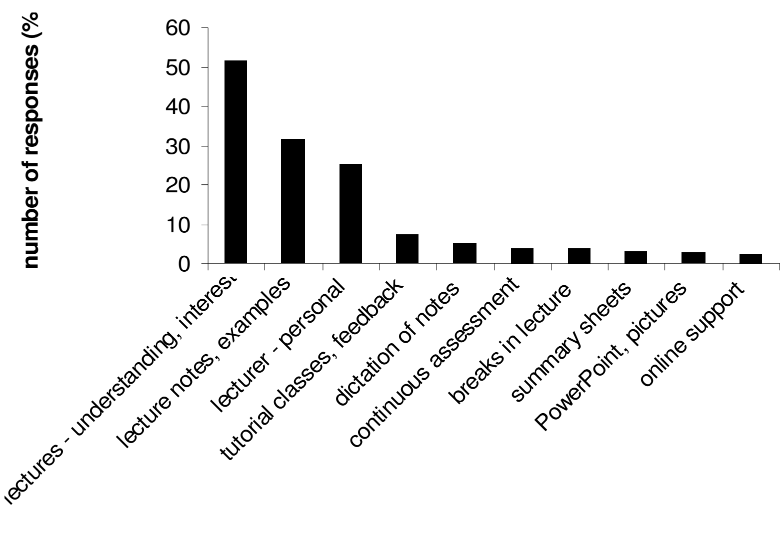

Figure 4: Student responses when asked to comment on the most satisfactory aspects of the module (2008 – 2011).

- **Lectures** – includes comments relating to lecture structure and relevance, also lecture being informative, interesting and easy to follow

"The clarity of the lectures and the handy summary sheets. Building confidence in maths."
"Explained how we could be using the maths practically in the real world."
"Note taking helped focus throughout the class."
"Regular breaks helped keep concentration in classes."
"Slightly informal so more enjoyable."
"Made learning interesting and being in class enjoyable."
"Used visual aids very well, made the course easily understandable."

One theme which emerged very strongly from the focus groups, although not often stated explicitly on questionnaires, was that students prefer to write their notes during lectures rather than receive handouts of the lecture material. They believe this helps them stay focused, concentrating on the topic as they write. It was believed learning would be greater by this approach rather than by listening and reading the maths on the handout. Moreover, it was stated that for example questions in maths, it was important to implement the good practice of writing the complete solution, setting out all the steps, and that printed notes with blanks left for completing part of the solutions during the lecture were inadequate. It is notable that these points were particularly stressed by the second year interviewees whose opinions are given in the context of their extra year's learning experience.

Active learning [7] and student participation during lectures are further encouraged by the lecturer asking questions. Asking students to vote on various possible answers to a question promotes student participation without the pressure of speaking in front of a large group.

PowerPoint is applied in lectures, for example, to develop a result term by term. Likewise, graphs can be constructed gradually and imaginatively. This helps retain students' interest while presentation is enhanced. Some focus group participants emphasised that the visual aspect of a lecture was memorable and useful in attracting their attention to an important result.

- **Lecture notes and examples**

"Well structured notes. Very good for going back and revising."
"Notes and examples were very clear and well organised and easy to understand."
"Use of a lot of examples in lectures makes it easier to understand what is being taught."
"Because of the layout my understanding of maths has greatly improved."

Much effort has been made to present the material clearly and logically so that students are able to follow their notes after the lecture. Numbering sections in the lecture notes, eg 1.1, 1.2, 1.2.1, emphasises the structure and separates different topics. Numerous worked examples are included to help reinforce the material being taught and allow demonstration of potential pitfalls in the solutions. Typical engineering applications are incorporated to illustrate the usefulness of mathematics to engineers – this makes the lectures more interesting. Summary sheets containing key results are provided at the conclusion of major topics to help students extract the main points.

Another strong theme coming from the focus group was that students appreciate some extra background theory and also examples of where the maths can be applied in engineering, believing this to be important for their learning. At school, students were provided with formulas to be memorised and were shown how to do questions. They believed that having the theory, seeing where formulas came from, and deriving results through a step-by-step method would aid their learning and understanding. This deeper approach would be helpful when they apply the maths in other modules such as engineering dynamics.

- **Lecturer** – comments more related to the person, delivery style, interaction, approachability

"A motivated teacher who made learning interesting and being in class enjoyable. The lecturer's interest in teaching."
"Lecturing style was very engaging and helped to make what is normally a dull subject matter actually exciting."
"Instead of being lectured at you felt that you were being taught."

Student feedback indicates that an enthusiastic and personal approach, establishing a good rapport with students, is significant. Many students appreciate humour and some lighter moments interspersing the detailed mathematics within the lectures. The lecturer often attends tutorial classes and talks to the students, showing interest in their progress. He also makes the effort to learn students' names and this is often remarked on by the students. This method should mean students will find it easier to ask questions and it is interesting that students make a distinction between a lecturer and a teacher.

- **Mid-term class tests**

"Class tests encourage learning as we go through the course rather than leaving it all to the final exam."

These are generally popular and students tend to take them seriously. Students recognise that tests keep them engaged and encourage them to work steadily over the semester. They readily admit that without such assessment, they would not be motivated to study outside of class. It has been observed that students, many of whom have part-time jobs, allocate time strategically, focusing on what is assessed and spending little time on unassessed tasks [8]. Tests have additional advantages in that a good performance in the tests increases students' confidence while weak areas can be identified and targeted for extra revision before the final exam.

In Further Mathematics 1, tests normally occur in weeks 5 and 10, each counting for 10% of the module mark. Until 2007, the tests and exam required the students to present their worked solutions – method marks were available. A very strong correlation between the marks achieved in the tests and the final exam was observed (typical correlation coefficient, r = 0.80). Since 2008, the tests consisted entirely of multiple choice questions (no method marks available) and the exam contained a mixture of multiple choice and structured questions. The format was changed to reduce the workload for the assessor and permit quicker feedback. A strong correlation still existed between test and exam marks (Figure 5) although the correlation was weaker than before (typical r = 0.64). This suggests that the multiple choice test is not as good a predictor of exam performance. There is a risk that this form of test, and the absence of method marks, would penalise weaker students but care was taken in setting the questions to minimise this. For example, the solution of a differential equation could be broken into two steps with a separate question on each step.

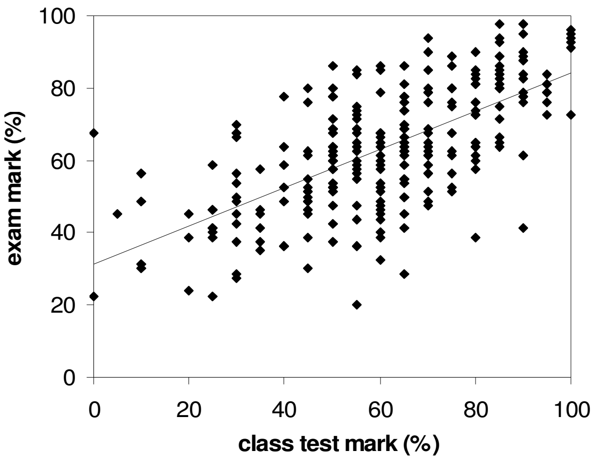

Figure 5: Exam and class test marks for the students taking Further Mathematics 1 in 2010.

- **Tutorial / exercise classes**

"Plenty of opportunity to ask questions and resolve problems in the tutorials."
"Feedback one on one was available and very useful."
"Good use of Queen's Online for tutorials and past papers."

In these more informal classes, the students work through practice questions with help available from the lecturer or postgraduate assistant. Answers are attached with the questions while worked solutions are provided on the University's intranet a few days after the tutorial. Group working enables learning through an explanation, or by reading a worked solution, of another student – this is not discouraged.

The importance of the tutorial class accommodation was highlighted during one student interview. Tutorials should not occur in the more confined lecture theatre setting; they should use a room with larger tables where students have more space to set out their notes and consult with others. A single page set of questions, getting gradually more difficult, each week was deemed to be sufficient – too many questions each week could be daunting or tedious.

The focus group indicated that the most effective resources for learning were the lecture notes (and examples) and associated tutorial questions. Past exam papers would be used heavily in preparation for the exam. There didn't seem to be much interest in using online resources other than those provided by the lecturer on the module intranet site.

Figure 6 shows the number of downloads of tutorial solutions each day over the semester. The peaks in usage correspond to the assessments – the two class tests and exam. The largest usage occurred on the day before the exam (412 downloads on 24 May 2010) – this point has been omitted from the graph to improve its clarity. Given the class size (250 students) and the fact that some students make multiple downloads of the same resource, use of tutorial solutions is generally low. After an initial burst of activity (initial keenness), resource usage dropped after the first class test and remained very low until the second test approached. Some students would be capable to completing the questions without consulting solutions. Perhaps others believed they progressed enough during the classes and didn't need to follow up with extra study at home. This observation ties in with the earlier comment about students spending little time on unassessed tasks [8].

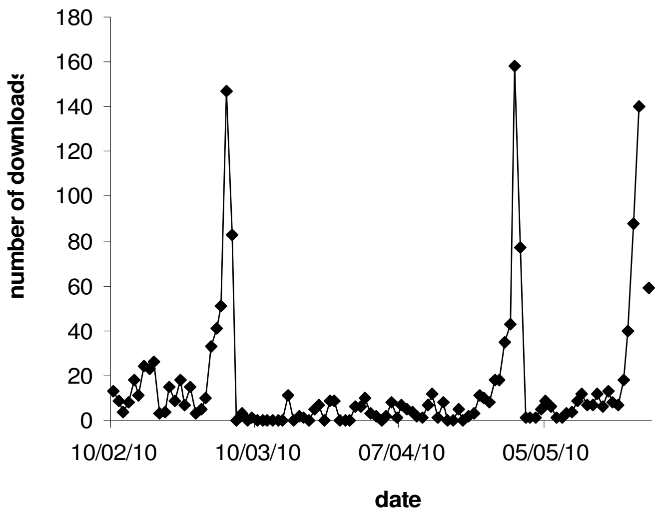

Figure 6: Daily number of downloads of tutorial solutions over the semester (2010).

3. Further Mathematics 2

At Stage 2, in the first semester Mathematics & Computing course, students are introduced to Matlab and learn some basic operations and functions. This is built upon in the second semester Further Mathematics 2 course where more applied mathematics topics are presented and application of the computing tools to aerospace problems is explored. Further Maths 2 has existed in its current form for four years. It is compulsory for second year aerospace engineering students enrolled on the MEng programme – typically 20 students each year.

Class time consists of 24 hours of maths teaching (over 12 weeks) and 15 hours of Matlab computing (a 1-hour lecture followed by a 2-hour laboratory each week for five weeks). The maths is assessed by a 2-hour exam contributing towards 80% of the course mark while the computing is assessed continuously.

The maths content involves more applied topics including linear algebra, multiple integrals, vector calculus and Laplace transforms. Examples relevant to aerodynamics, structural analysis and engineering dynamics are presented. The aim is to develop a sound understanding and practice of essential mathematics tools used in engineering.

The first semester computing course involves the basic workings and functions of Matlab. For almost all students, this is probably their first introduction to Matlab. The computing in Further Maths 2 follows on from this and aims to encourage a systematic approach to problem solving and apply the maths and computing tools to aerospace engineering.

3.1 Student Learning Experience

Student performance in both the maths and computing sections of the course has been good with typical average marks of 65% and 70% respectively. This level of achievement is to be expected of MEng students.

Student comments showed that the learning experience was enhanced through demonstrating the relevance of the maths.

> *"Really helps for aerodynamics and parts of propulsion."*
> *"Clear link between maths being taught and the other modules, especially aerodynamics."*
> *"Greater ability to apply maths to other modules as aside from just within its own module."*

The students seemed to enjoy the style of teaching. The 2-hour lecture slot each week contains a combination of formal teaching and student work on example questions. The latter activity is introduced as appropriate, for example at the end of a topic, and students have opportunity to practise what has been taught. This keeps students alert and allows feedback (both for themselves and the lecturer) on their progress. Having a mixture of activities during the class is also advantageous in that it simply breaks up the 2-hour session, helping to maintain student attention and involvement. Even during the formal teaching sessions, student participation occurs with students asked for answers as a worked solution is developed. This can be achieved relatively painlessly given the small class size and the fact that the students know each other very well.

> *"Breaks and tutorial questions help break up session and keep my attention span."*
> *"Good explanations and examples to reinforce information being taught."*
> *"Clear and concise, structured, good support for examples."*
> *"Good, involving, how teaching should be."*

The relatively high rate of attendance at classes (average 77% in recent years) gives evidence of an enhanced learning environment.

Students found aspects of the computing to be difficult. However, they were able to identify personal gains from the course. It is interesting that some focused on subject-specific skills while others mentioned transferable skills.

> *"Ability to more clearly structure my approach to problems and be more methodical in solving them."*
> *"Troubleshooting skills, breaking down engineering concepts into Matlab code."*
> *"Understanding of practical uses for Matlab."*
> *"Better understanding of how to apply Matlab commands."*
> *"How to analyse and work through more complex problem."*

"A better understanding of Matlab. Second part allowed for better and useful programmes to be written."

It is suggested that positive aspects of the teaching methodology include the gradual building of computing skills and using a range of aerospace applications. Students' skills are built up in the early weeks as the course covers loops, functions, writing to files, good programming practice, etc. These skills are then applied to aerospace engineering problems, for example, estimating the lift on a wing. Some time is spent manipulating matrices, solving sets of equations and calculating eigenvalues to help understanding of these topics which also occur in the maths section. There is plenty of interaction between lecturer and students in the lab sessions with students able to receive individual attention.

A variety of responses was received concerning the relevance of the computing section. Most students noted the aerospace applications and links to other modules in the degree programme – structures, aerodynamics and design were mentioned. A few were not convinced that these links were of much significance. Interestingly, one student stated that the main relevance for him was that it prompted an improvement in his approach to problem solving in general.

"Tied into a number of modules both past and present."
"All applications were aerospace orientated."
"It changed my approach to problems. I feel more comfortable in my approach to problems in any subject."

4. Conclusions

This paper has reported the author's experiences of teaching mathematics over ten years to diverse groups of engineering students. Student responses on module evaluation questionnaires and focus group interviews have been used to identify aspects of teaching which contribute to an enhanced learning experience.

A first year mathematics module has been provided to large classes of aerospace, chemical, civil and mechanical engineering students (typical class size is 250). It aims to provide students with a good grounding in a range of fundamental topics relevant to engineering. Exam results and pass rates have been pleasing and relatively consistent. The pass rate for the course is 89% while the exam average mark is 65% (mean values over nine years).

Student feedback indicates a preference to write notes during lectures rather than receive handouts – this helps them focus and it also teaches the good practice of setting out a solution sequentially. Students believe that, rather than being presented with formulas to learn, having some background theory and deriving results through a step-by-step method aids their learning and understanding. This approach would also be helpful when they apply the maths in other modules. Clear presentation of material, having plenty of good examples, and demonstrating application of the maths are also important for learning. Students appreciate mid-term class tests, understanding that they promote a more structured approach to learning. The most effective resources for learning were said to be the lecture notes (and examples) and associated tutorial questions. Past exam papers are used heavily in preparation for the exam.

The second year aerospace engineering MEng students (class size 20) take a mathematics and computing module. The maths section involves more applied topics such as linear algebra, vector calculus and Laplace transforms. The small class size has allowed a more interactive approach and a mixture of formal teaching and student practice within the sessions. Student feedback suggests they enjoy the involving style of teaching. The computing section, based on Matlab, aims to demonstrate application of the maths and show the usefulness of computing tools in aerospace engineering. Students have found the Matlab section to be challenging but could identify gains from the course, both in terms of improved programming ability and learning a more methodical approach to problem solving.

In general, student feedback indicates that an enthusiastic and personal approach, establishing a good rapport with students, is significant. This requires attending tutorial classes, talking to students, encouraging them, even trying to learn their names. It is also interesting to note that students make a distinction between a lecturer and a teacher.

5. Acknowledgements

The author wishes to thank the following students for their contribution in preparing this paper: Jonathan Adams, Hannah Beattie, Jason Bloomfield, Fiona Boyle, Julie Forbes, Alastair Martin, Peter Mathews, John McClelland, Aislinn McGinn, Leigh Robinson.

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