A Simple Experimental Procedure for the Determination of the Heat Capacity of Metals under the Premises of a Lumped Model and Natural Convective Cooling

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Abstract — An experiment has been conceived to determine the specific heat capacity of pure metals and alloys. In this regard, a small-diameter metallic sphere at ambient temperature (around 20°C) is suddenly dropped in a large tank containing quiescent iced cold water at 0°C. The metallic sphere is suspended from the ceiling of a large room by the lead wire of the thermocouple. The modeling is done with a nonlinear differential equation of first order called Bernoulli equation. For aluminum alloy the error of the measured specific heat capacity is within a band of 20% error.

Index Terms — modeling, nonlinear natural convection, specific heat capacity, experimental measurements

NOMENCLATURE

\[ a, b \] coefficients in (9)
\[ A \] surface area of sphere, \( \text{m}^2 \)
\[ Bi \] Biot number for a lumped sphere, \( hR/3k \), dimensionless
\[ c_p \] specific heat capacity at constant pressure, \( \text{J/kg.C} \)
\[ c_v \] specific heat capacity at constant volume, \( \text{J/kg.C} \)
\[ D \] diameter of sphere, \( \text{m} \)
\[ g \] gravitational acceleration, \( \text{m/s}^2 \)
\[ Gr \] Grashof number, \( (g \beta \nu^2)(T - T_\infty)D^3 \), dimensionless
\[ h \] mean heat transfer coefficient, \( \text{W/m}^2.\text{C} \)
\[ k \] thermal conductivity, \( \text{W/m.C} \)
\[ Nu \] dimensionless measured temperature
\[ NuD \] mean Nusselt number, \( hD/kf \), dimensionless
\[ Pr \] Prandtl number, \( \nu/c_p/k \), dimensionless
\[ R \] radius of sphere, \( \text{m} \)
\[ s \] characteristic length, \( \text{V/A, m} \)
\[ t \] time, \( \text{s} \)
\[ T \] space-mean temperature, \( \text{C} \)
\[ T_\infty \] free-stream temperature of water, \( \text{C} \)
\[ V \] volume of sphere, \( \text{m}^3 \)

Greek Letters

\[ \lambda \] thermal diffusivity, \( k/c_p \), \( \text{m}^2/\text{s} \)
\[ \alpha \] coefficient of volumetric thermal expansion, \( 1/\text{K} \)
\[ \Delta T \] temperature difference, \( T - T_\infty \), \( \text{C} \)
\[ \mu \] dynamic viscosity, \( \text{N.s/m}^2 \)
\[ \nu \] kinematic viscosity, \( \text{m}^2/\text{s} \)
\[ \rho \] density, \( \text{kg/m}^3 \)
**INTRODUCTION**

It may be recalled from Thermodynamics that the specific heat capacity is defined as the energy required to raise the temperature of a substance by one degree per unit mass (Moran and Shapiro [1]). Based on this, there are two types of specific heat capacities, one at constant pressure \( c_p \) and the other at constant volume \( c_v \):

\[
\begin{align*}
    c_p &= \left( \frac{\partial i}{\partial T} \right)_p, \\
    c_v &= \left( \frac{\partial u}{\partial T} \right)_v,
\end{align*}
\]

where \( i \) is the specific enthalpy, \( u \) is the specific internal energy and \( T \) is the temperature. The subscripts \( p \) and \( v \) in the symbol \( c \) denote constant pressure and constant volume, respectively.

It has been demonstrated both theoretically and experimentally that for gases, \( c_p \) is always larger than \( c_v \), whereas for solids and incompressible liquids \( c_p \) is equal to \( c_v \).

The specific heat at constant pressure \( c_p \) is used extensively in unsteady heat conduction because this property is one of the three properties that conform the thermal diffusivity \( \alpha \):

\[
\alpha = \frac{k}{\rho c_p}.
\]

Once \( \alpha \), \( k \) and \( c_p \) are measured, \( \alpha \) may be readily calculated from the preceding relation.

As is the case with thermal conductivity \( k \), values of \( c_p \) for numerous materials for engineering applications have been published in various handbooks (see for instance Touloukian et al. [2]), but as new materials are developed, it is important for students in mechanical, chemical engineering and other allied disciplines to be familiar with fundamental methods of measuring \( k \) and \( c_p \).

In general, the measurement of the specific heat \( c_p \) belongs to a class of experimental methods called calorimetry [1]. There are different calorimetry techniques for the measurement of specific heat. Which of these techniques is utilized in the laboratory depends on the type of material to be tested, the temperature range, the accuracy and other intervening factors.

Alternatively, \( c_p \) may be measured in an undergraduate thermofluids laboratory using simple concepts from unsteady heat conduction (Kreith and Bohn [3]). Essentially, this approach constitutes the central objective of this paper on engineering education.

**THEORETICAL BACKGROUND**

In order to determine the specific heat capacity of pure metals and alloys, an experiment has been conceived for a situation in which a small-diameter metallic sphere at ambient temperature (around 20 \( ^\circ \)C) is suddenly dropped in a large tank containing quiescent iced cold water at 0 \( ^\circ \)C. The metallic sphere is suspended from the ceiling of a large room by the lead wire of the thermocouple. For simplicity, the physical properties of both the metal and water are taken as constant in the 20 \( ^\circ \)C temperature interval.

From physical concepts, it may be inferred that the heat removal from the metallic sphere to iced cold water occurs by natural convection; this process being dependent on time. In this regard, the sphere's surface was carefully polished in advance in order to minimize the radiative transfer to the coolant during the cooling period.

To avoid the presence of the heat conduction inside the sphere, the magnitude of the Biot number \( Bi = hR/(3k) \) has to be less than 0.1 [3]. Accordingly, the applicable lumped energy equation, along with its initial condition, is expressed as

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Next, introducing the volume-to-area ratio of a sphere V/A = D/6, this equation becomes

$$\rho_c V \frac{dT}{dt} = -\bar{h} A (T - T_\infty), \quad T(0) = T_0. \quad (1)$$

Dimensional analysis of the physical variables affecting the heat dissipation by natural convection from a solid body to a
Newtonian viscous fluid establishes a well known functional relation between the Nusselt number NuD, the Grashof number GrD
and the Prandtl number Pr [3]. Several years ago, Yuge [4] gathered experimental data for constant temperature spheres cooled in
regular viscous fluids and was able to convert the function $\text{Nu}_D = f(\text{Gr}_D, \text{Pr})$ into an empirical correlation equation of power form

$$\text{Nu}_D = 2 + 0.43 \left( \text{Gr}_D \text{Pr} \right)^{1/4} \quad (3)$$

The domain validity of this equation is for $\text{Gr}_D \text{Pr} \leq 10^5$ with regular fluids. Another useful correlation equation that surpasses
$\text{Gr}_D \text{Pr} = 10^5$ was constructed by Churchill [5]:

$$\text{Nu}_D = 2 + 0.589 \left( \frac{\text{Gr}_D \text{Pr}}{f(Pr)} \right)^{1/4} \quad (4a)$$

where $f(Pr)$ stands for a so-called 'universal' Prandtl number function

$$f(Pr) = \left[ 1 + \left( \frac{0.469 \text{Pr}^{1/4}}{f(Pr)} \right)^{1/4} \right]^{4/9} \quad (4b)$$

The domain of validity of (4) covers a large interval $0 < \text{Gr}_D \text{Pr} < 10^{11}$ for the spectrum of nonmetallic fluids with $\text{Pr} \geq 0.7$.

From the standpoint of heat transfer analysis, the sphere constitutes an interesting configuration because the mean Nusselt
number $\text{Nu}_D$ in (4) has a clear specifiable lower bound $\text{Nu}_D = hD/k_0 \to 2$ as the ratio of buoyant to viscous forces vanishes for $\text{Gr}_D \to 0$. So, in this lower limit the transfer of heat from the sphere to the neighboring fluid is solely controlled by heat conduction.

From a different perspective, the behavior of the convective heat transfer coefficient $h$ for the sphere surrounded by quiescent
iced cold water as $h$ is affected by the influential geometric, hydrodynamic and thermal quantities in (4). This step, crucial to the
fulfillment of the goals of this paper, leads to the following two-term expression for $h$:

$$\text{Pr} = 2 \nu \beta + 0.413 \frac{k_0}{D} \left( \frac{g \beta}{\nu^2} \right)^{1/4} \left( \nu \beta \rho \right)^{1/4} \left( T - T_\infty \right)^{1/4} \quad (5)$$

where for convenience $g(Pr) = Pr^{1/4}/f(Pr)$. It is evident at this point that once the diameter of the sphere $D$ is specified and the two
properties of the iced cold water $k_0$ and $\nu$ have been evaluated, $h$ deviates from being a constant quantity. On the contrary, $h$
emerges as a variable quantity which is vulnerable to the instantaneous temperature of the sphere $T$ through the nonlinear
temperature function $(T - T_\infty)^{1/4}$. Further, since $T$ varies with time, the transitive rule indicates that $h$ varies with time too during
the cooling period. It should be stressed that the function $(T - T_\infty)^{1/4}$ is restricted to laminar natural convection flows exclusively.

Introduction of (5) into (2) delivers the non-homogeneous differential equation

$$\rho_c D \frac{dT}{dt} = -12 \frac{k_0}{D} (T - T_\infty)^{1/4} - 2.478 \frac{k_0}{D^{3/4}} \frac{g \beta}{\nu} \left( \nu \beta \rho \right)^{1/4} \left( T - T_\infty \right)^{1/4} \quad (6)$$

which is mildly nonlinear and, in all likelihood, does not allow for an analytic solution. However, adoption of the temperature
difference transformation

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is beneficial because it homogenizes (6). As a consequence of this, the new differential equation and the initial condition are
\[
\frac{d\theta}{dt} + a \theta + b \theta^{5/4} = 0, \quad \theta(0) = T_0 - T_\infty.
\] (8)

The participating coefficients \(a\) and \(b\) vary inversely with the diameter \(D\) of the sphere and are computed from the ratios:
\[
a = \frac{12 k_0}{\rho c} \frac{1}{D}, \quad b = \frac{2.478 k_0}{\rho c} \left( \frac{g \beta}{v^2} \right) \left( \frac{g(Pr_\infty)}{D^{5/4}} \right) \frac{1}{D^{3/4}}.
\] (9)

The resulting first-order differential equation (8) is nonlinear and is known as the Bernoulli equation in the mathematical literature (Polyanin and Zaitsev [6]). The corollary that surfaces up from the preceding statements is that the time-dependent natural convection is a nonlinear mode because \(h\) changes with \(T\) in a nonlinear fashion for a fixed lapse of time.

Luckily (8) can be solved via a variable transformation. Accordingly, letting \(u = \theta^{1/4}\), the nonlinear equation (8) is converted into a linear equation in the new variable \(u\). The new equation is then amenable to the method of separation of variables (Ince [7]). In the end, the exact analytic solution of (8) turns out to be
\[
\frac{T(t) - T_\infty}{T_0 - T_\infty} = \left[ 1 + \frac{b}{a} \exp \left( \frac{at}{\sqrt{4}} \right) \right] \frac{b}{a}.
\] (10)

This equation provides the dimensionless temperature distribution for the cooling of the sphere.

A pleasant surprise of the mathematical analysis is that within the framework of natural convection, the temperature vs. time variation of the sphere may be predicted in exact form from the initial temperature \(T_0\) until the ambient temperature \(T_\infty\) is reached for a long time, in principle \(t \to \infty\) (a state of thermodynamic equilibrium).

**EXPERIMENTAL MEASUREMENTS**

The experimental setup consisted of several metallic spheres of 1 inch in diameter made from aluminum alloy 2017-T451, T-type thermocouples, a medium size bucket, a bag of crushed ice and a computer system to acquire and record the instantaneous temperature vs. time data. One thermocouple was attached to the sphere's surface with Omegabond OB-101 conductive epoxy adhesive. It was expected that the very delicate attachment was capable of preventing significant heat conduction through the thermocouple wire. Previously, the thermocouple was calibrated in an ice water bath held at 0°C.

A simple arrangement allowed the sphere to be suspended from the thermocouple while in contact with the iced water. Temperature vs. time data from the thermocouple was monitored through an Analog Devices digital thermometer #AD2051. The amplified signal from the digital thermometer was fed into a National Instruments #PC516 data acquisition board through a Labview Virtual Interface. Once the sphere was hanging motionless, the temperatures of the thermocouple were recorded at a rate of 2 readings per second. The sphere was allowed to cool without stirring the bath for approximately 90 seconds or until the final temperature of the sphere reached steady state conditions at 0°C.

Air movement in the room was avoided during testing to reduce any forced convection air currents.

The properties of the aluminum alloy at an ambient temperature of 20°C are: density \(\rho = 2.79\) gm/cc, specific heat capacity \(c_v = 0.88\) J/g.C, and thermal conductivity \(k_0 = 134\) W/m.K.

**DETERMINATION OF THE SPECIFIC HEAT CAPACITY**

The properties of the iced water are customarily evaluated at the film temperature of 10°C: the thermal conductivity \(k_w = 0.577\) W/m.C, the property ratio \((g \beta / v^2)^{1/4} = 0.551 \times 10^9\), and the combined 'universal' Prandtl number function \(g(Pr_\infty) = 1.63\).

In compliance with the lumped model, the Biot number for a lumped sphere \(Bi = hR/3k_w\) must be calculated first. Since \(h\)}
exhibits a monotonic decreasing behavior with increments in time, the initial heat transfer coefficient \( h_i \) at \( t = 0 \) corresponds to the largest \( h \) in the entire cooling process. Accordingly, \( h_i \) is evaluated from (5) giving \( h_i = 506 \text{ W/m}^2\text{K} \). The resulting initial \( Bi = 0.05 \) being less than 0.1 warrants the utilization of the lumped model.

The contribution of radiation transfer needs to addressed also. For a polished aluminum alloy the total hemispherical emissivity is \( = 0.05 \) [3]. The initial value of the radiation heat transfer coefficient \( h_{r,i} \) is 0.26 W/m\(^2\)K. Clearly, this number is minuscule when compared against the initial heat transfer coefficient \( h_i = 506 \text{ W/m}^2\text{K} \).

For purposes of estimating the specific heat capacity \( c_s \), it is convenient to form first the coefficient ratio \( c = b/a \) resulting in

\[
c = \frac{b}{a} = 0.295 \left( \frac{g \beta}{V} \right)^{1/4} \frac{g(Pr)}{D^{3/4}} \tag{11}\]

Next, solving for the coefficient \( a \) in (10) gives

\[
a = \frac{4}{t_m} \ln \left( \frac{1 + N_a c - N_a}{N_a c} \right) \tag{12}\]

The measured temperature \( T_m \) at the corresponding time \( t_m \) which can be taken from the recorded data. With this information, the dimensionless temperature ratio \( N_m \) may be obtained at the real time \( t_m \)

\[
N_m = \left( \frac{T_m - T_i}{T_i - T_0} \right)^{1/4}, \; t_m
\]

Using eq. (9), we can immediately write the predictive expression for \( c_s \)

\[
c_s = \frac{12 k_c}{a \rho_i} \cdot \frac{1}{D^2} \tag{13}\]

where the value of \( a \) comes from (12).

Fig. 1 illustrates the variation of the measured instantaneous temperature with time starting with an initial temperature of 22 C. We chose the pair of points \( T_m = 4 \text{ C} \) and \( t_m = 80 \text{ s} \) that results in \( N_m = 0.653 \). Knowing that the coefficient \( c = 4.69 \) from (11), the coefficient \( a = 0.0054 \) is computed from (12). From here, \( c_s = 712.55 \text{ J/kg.C} \) is readily estimated from (13).

When the estimated \( c_s \) from the experiment is compared with the real \( c_s = 880 \text{ J/kg.C} \) supplied by the manufacturer of the aluminum alloy spheres, the error is of the order of 19%. This error seems to be reasonable taking into consideration the simplicity of the experiment and the relatively small temperature differential of 20 C employed to induce the upflow natural convection currents.

Future work will be centered on the use of a larger temperature differential around 100 C which is associated with the heating of the sphere in a bath of boiling water at 100 C.

REFERENCES


FIGURES AND TABLES

FIGURE 1
TEMPERATURE – TIME HISTORY

Cooling Response of a 1" Aluminum Sphere