INVESTIGATIVE PROJECTS IN ENGINEERING FOR SECONDARY SCHOOL STUDENTS

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Abstract *¾ This paper describes a series of activities for the senior secondary mathematics class room which have been developed to illustrate how mathematics is used by engineers. In one example students are presented with an innovative design for a single storey underground car park. The class room activity based upon the car park design covers topics such as volumes of revolution, fitting of hyperbolic functions and length of arc calculations. Another example considers the pasteurisation of cream. Students are presented with real physical property data for creams of different fat contents and are then asked to calculate the amount of energy needed to raise the temperature of the cream to a specified temperature for pasteurisation purposes. This activity covers curve fitting to x-y data and integration and introduces the students to chemical and bioprocess engineering concepts.*

Index Terms ¾ mathematics, outreach, secondary school students

INTRODUCTION

The community in general has a very good appreciation of many of the professions vital to today's society. Through television and the wider media people generally know what activities doctors and lawyers, policemen and dentists perform in their day to day activities. The same cannot be said for the engineering profession however. The popular image of the professional engineer is a man who wears a hard hat and builds bridges, tunnels and tall buildings. This inaccurate perception does not recognise the increasing number of women engineers nor the range of diverse engineering disciplines including chemical, electrical, environmental, mechanical, petroleum and software engineering. If this is true for the general community it is certainly true for the secondary school community. Students and teachers alike usually do not have an appreciation of the diversity of challenges and opportunities that await them in an engineering career.

While any secondary school student wishing to enter a university engineering program must have a high degree of competency in mathematics, most secondary school mathematics teachers do not have an appreciation for the engineering discipline. Whether they are teaching simple algebra or more advanced calculus, mathematics teachers at any year level are often confronted by students with questions such as “Why are we learning this?” and “What is this used for?” Often the teachers do not have the background to be able to answer these questions adequately. And, just as importantly, the teachers want examples of real life applications of mathematics available to them.

While there is a clear need to raise the profile of the engineering profession in the secondary school community there are several ways to go about doing this. One strategy tried by many institutions and professional bodies has been to target secondary school students. By either sending academics out into the schools or by bringing school groups into the universities a limited number of potential engineering students become aware of the profession. The disadvantage with this strategy is that as the students advance through schools it is necessary to repeat these activities each year to a different cohort of students. A more efficient option is to work with teachers in programs which introduce them to engineering concepts in a way that is both interesting and of practical use in the class room.

When considering possible careers, secondary school students are often heavily influenced by their teachers. A teacher with a misconception of the engineering profession could, over their own career, encourage a generation of students to choose careers other than engineering. In some cases teachers have been known to actively discourage their brighter students from studying engineering. This is often particularly true at some single-sex girls schools in which teachers are unaware of the rising proportion of women working in the engineering profession.

Mathematics is the science that underpins all engineering. Good engineers must have superior mathematical skills and a sound grounding in analytical principles. Recognising this in recent years the Faculty of Engineering at the University of Melbourne has begun working with the local mathematics teachers association, the Mathematical Association of Victoria, to develop mathematics exercises which show how mathematics is used by engineers to solve real problems. Teachers and students alike appreciate and enjoy working with these real problems which demonstrate aspects of engineering design and problem-solving skills.

In 1994 the author began developing a series of class room activities based around the design of a bulk liquid chemical storage facility or tank farm [1]. Since that time professional development sessions for secondary schools teachers based on the activity have been conducted in Australia, Indonesia, Malaysia, Thailand and Vietnam. In 1999 the author, working with four practicing mathematics

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teachers published a book which presents further problems based around the tank farm activity [2]. The book includes a selection of practical ideas for use in the class room. The activities range from simple exercises that may be completed in minutes to week-long class projects.

This paper describes a series of activities for the senior secondary mathematics class room which have been developed to illustrate how mathematics is used by engineers. In one example students consider the pasteurisation of cream. Students are presented with real physical property data for creams of different fat contents and are then asked to calculate the amount of energy needed to raise the temperature of the cream to a specified temperature for pasteurisation purposes. In another example students are presented with an innovative design for an underground car park which permits trees in the landscaped garden above to grow into cavities above the main columns. The class room activity based upon the car park design covers topics such as volumes of revolution, fitting of hyperbolic functions and length of arc calculations. A third activity introduces the concepts of process control and allows students to simulate the control of a simple process using a spreadsheet and knowledge of simple calculus.

PASTEURISATION OF CREAM

The following example was presented to teachers in 2000 to illustrate how data for a particular situation presented in tabular and graphical form may be modelled and analysed using functions and calculus [3]. In presenting the material it was necessary to provide the teachers background information on both the pasteurisation process as well as the concept of specific heat capacity. In preparing the material the role of the engineer is clearly stated.

Fresh, untreated cows milk contains micro-organisms that can be harmful to humans. In order to kill these micro-organisms the milk is heated to a temperature around 75°C and kept at that temperature briefly before being cooled. This process is known as pasteurisation. All dairy products including skim milk and cream must be processed in this way.

In heating the dairy products care must be taken to ensure that the specified temperature is obtained. If too little heat is applied to the milk or cream then the temperature will not be hot enough to kill all the microorganisms, while if too much heat is applied the temperature will be too hot, degrading the quality of the milk or cream.

Engineers designing the equipment which heats and then cools the milk must know the amount the energy that must be applied to heat the milk or cream to the right temperature.

- The amount of energy required to heat a liquid such as cream depends upon three factors:
- the amount of cream required to be heated;
- the extent to which the temperature is to be increased; and
- a physical properry of the cream known as its specific heat capacity.

The specific heat capacity of a liquid such as cream is the amount of energy measured in kJ required to raise 1 kg by 1ºC. Specific heat capacity is measured in units of kJ/kgºC. For example, for cream with a fat content of 60 % the specific heat capacity at 10ºC is 3.90 kJ/kgºC. This means that to raise the temperature of 1 kg of cream by 1ºC, 3.90 kJ of energy must be applied.

The calculation of the amount of energy required to heat 1 kg of cream not 1ºC but 10ºC or perhaps 50ºC is complicated by the fact that the specific heat of cream is a function of temperature.

Table I shows the values for the specific heat capacity of three different types of cream. This data is real.

TABLE I
SPECIFIC HEAT CAPACITY OF CREAMS

Temperature (°C)	20 % Fat Cream (kJ/kg °C)	40 % Fat Cream (kJ/kg °C)	60 % Fat Cream (kJ/kg °C)
0	3.62	3.33	2.43
2	3.71	3.46	2.70
4	3.77	3.60	3.02
6	3.83	3.73	3.35
8	3.86	3.87	3.61
10	3.87	4.00	3.88
12	3.91	4.14	4.19
14	3.94	4.27	4.42
16	4.00	4.39	4.33
18	4.15	4.41	3.92
20	4.18	4.38	3.59
25	3.88	3.68	3.29
30	3.79	3.67	3.18
35	3.73	3.63	3.12
40	3.66	3.30	3.08
45	3.65	3.26	3.05
50	3.60	3.24	3.04
55	3.61	3.21	3.02
60	3.63	3.15	3.03

Pasteurisation is one of the most important stages in the processing of dairy products. It helps to ensure that the products are safe to be consumed by the general public. In the pasteurisation process it is important that the dairy product be heated to exactly the right temperature, and held at that temperature for the correct amount of time, which may range from tens of seconds to several minutes. Chemical engineers who design the pasteurisation process equipment such as the heat exchanger used to heat and then cool the dairy product must understand the heating characteristics of the milk or cream. They must know how the dairy products' physical properties such as specific heat capacity, density, viscosity and thermal conductivity vary with temperature, and with the composition and nature of the dairy products. Engineers use this information to perform design calculations to design equipment that will meet the required specifications.

Mathematical Treatment

The data from the table is shown in graphical form in Figure 1. This data can be used as a basis for developing exercises based around three components:

1. Introduction of a context through specific cases or examples.
2. Consideration of general features of this context.
3. Variation, or further specification, of assumptions or conditions involved in the context to focus on a particular feature related to the context.

These components may be related to developing an activity based on the context of the pasteurisation of cream.

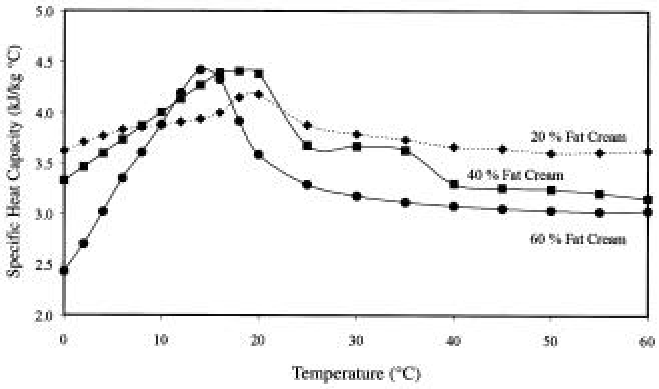

FIGURE 1
SPECIFIC HEAT CAPACITY OF THREE DIFFERENT CREAMS

Component 1

Using the data for one of the creams, for example, the 60% fat content cream, determine a single function or a hybrid function that models the data between 0ºC and 60ºC. For example, for 60% fat content cream, the data may be modelled by a straight line over the interval [0, 10], a parabola over the interval [10, 15] and an exponential function over the interval [15, 60]. This could be compared with a single function having a rule of the form:

$$s(t) = a\, t^{n} \times e^{-kt} + c \qquad (1)$$

for suitable values of the parameters a, n, k and c.

Component 2

In this section students could consider the construction of a suitable hybrid function in more detail. Different intervals or functions could be used, for example, for temperature up to about 12ºC to 14ºC for the 60% fat content cream, the data may be modelled by a straight line. For temperature between above about 12ºC and 14ºC and just below 20ºC a parabola might be used to fit the data, while a reciprocal function could be used for the rest of the domain. Whichever combination of intervals and functions is used, they should be joined smoothly where they meet. Similar models can be obtained for the other creams and compared with models for the 60% fat content cream.

Component 3

The functions used to develop various models can also be used to calculate the amount of energy required to heat cream from one temperature to another. This would involve the use of integration and approximation methods. Students could be asked to find the energy required for different creams and temperatures. Alternatively they could be asked to interpolate from the functions for these creams, and find a function for the specific heat capacity of cream having a fat content of 32 % at 50ºC.

Concluding Remarks

The pasteurisation of cream is one of many stages in the production of a variety of dairy products including drinking milk, cheese, skim milk powder and whey protein concentrate. Engineers participate in all stages of the design, construction and operation of dairy processing facilities. This example is a most useful one in the Australian context. Students are reminded that Australian dairy exports account for 12% of the worldwide trade in dairy products.

CONCRETE MUSHROOMS

An underground car park in Melbourne is one of the few modern structures on the Register of Historic Buildings. Constructed of reinforced concrete shells supported on short columns, viewed from within the car park it resembles a field of giant mushrooms. The innovative design permits landscaping above with trees growing within some of the soil cones present above each concrete column. Its design lends itself to some clever mathematical challenges, particularly for students in their senior years at secondary school.

Students are presented with a vertical cross-section through the axis of one of the concrete structures (Figure 2). The short cylindrical column or pier on which the upper concrete shell rests is clearly shown. Also shown is the conical well of soil into which trees may be planted.

The following example, developed by the author, is presenting being used in the State of Victoria in the senior secondary school mathematics subject, Specialist Mathematics. Again the activities are divided into three component.

Component 1

Specify the equations of all the straight lines defining the vertical cross-section of the concrete structure. Note that the height of the apex of the soil cone is not set. This must be set by the teacher and may be in the range from 1.00 m to 2.00 m. Specify in both Cartesian and parametric forms the equations defining the hyperbolic section. Note that at both ends of the hyperbolic curve the first derivative is continuous. What are the cross-sectional areas in the

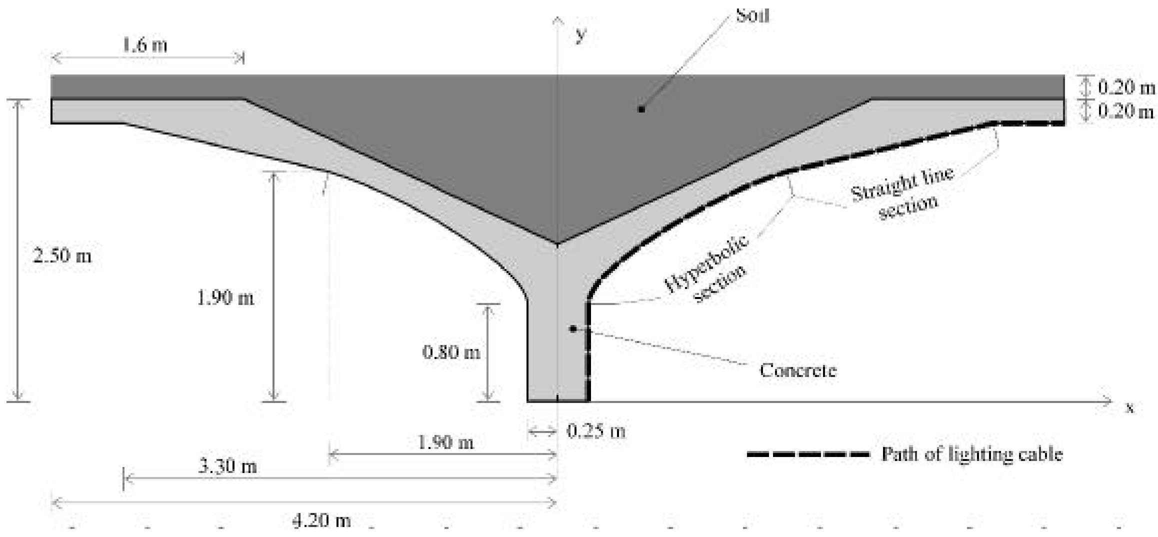

FIGURE 2
VERTICAL CROSS-SECTION OF THE ONE OF THE CAR PARK SUPPORT STRUCTURE SHOWING THE CONCRETE PIER AND SHELL

vertical plane through the axis of the columns of the concrete and of the soil ?

Component 2

Suppose that instead of the upper surface of the concrete shell being formed by a cone, it is formed by the rotation of a polynomial function of the form: $y = a + b\,x^{2n}$ where n is a series of positive integers beginning with 1. Derive an expression for the cross-sectional area of the concrete in terms of a, b and n for n=1 and 2.

Component 3

A cable is run along the underside of the concrete structure from its base to a point on the roof. This path is shown as a heavy dashed curve on the diagram. Find the length of the cable. Suppose that a shorter cable is attached at the base of the column and run up along the underside of the structure as shown. The cable will come to an end before it reaches the correct location of the light. Derive an expression which gives the height of the end point of the cable as a function of the length of the cable. Assume that one end of the cable is always attached to the base of the column.

KEEPING CONTROL

The topic of process control contains many examples of both simple and more advanced mathematics which may be used in the secondary school class room. The example which follows was first developed and presented in 1999 [4]. It shows how a spreadsheet package such as Microsoft Excel may be used to follow what happens to the temperature in a system when it is disturbed.

In introducing this to the class it is useful to talk about examples of control systems around the home. In Australia these include the control of the water level in the toilet, and temperature control in ovens, hot water systems and through central heating and air conditioning.

In the example presented in the senior class we talk about the process control responses of a system used to heat water flowing in a pipe. Let’s assume water at a temperature of 20ºC is flowing through a pipe at a rate of 0.25 litres per minute. Suppose we suddenly wish to increase the temperature of the water to some higher value, say 60ºC, by using an electrical heater. We might use a system as shown in Figure 3.

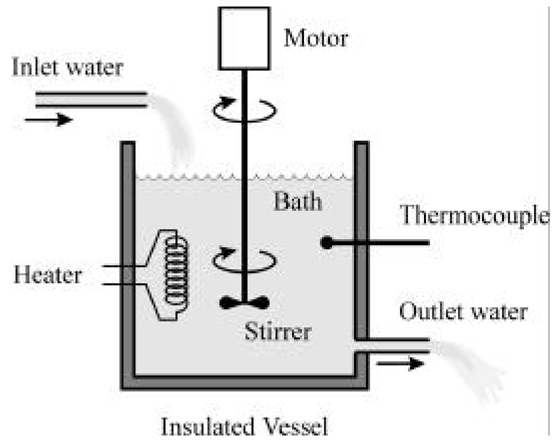

FIGURE 3
THE SYSTEM USED TO HEAT THE WATER FLOWING THROUGH A PIPE.

The water to be heated flows into a tank. If we assume that the water leaves the tank at the same flow rate as it enters then this means that the volume of water within the tank will remain constant. A stirrer inside the tank ensures that the water is well-mixed and that the temperature of the water is uniform throughout the tank. This also means that the temperature of the water inside the tank is equal to the water's exit temperature.

The water temperature is controlled using a system consisting of three components:

i) a thermocouple to measure the temperature of the water;
ii) an immersion heater to supply heat to the system; and
iii) a controller which controls the amount of heat sent to the heater in response to the temperature recorded by the thermocouple.

The signal from the thermocouple is transmitted to a controller. The controller compares the tank temperature to the desired value, known as the setpoint, which in this case is 60ºC and adjusts the amount of energy sent to the heating element.

The simplest controller is one in which the amount of energy supplied to the heating element, q, is directly proportional to the difference in temperature between the desired temperature, Ts, and the actual tank temperature, T. Thus,

$$q = K_c \left(T_s - T\right) \tag{2}$$

Here K_c is the proportional gain. This equation suggests that with T_s = 60ºC, if the water temperature in the tank is 40ºC the heater will put out twice as much energy as it would if the water temperature were to be 50ºC.

We now write an equation which balances all the energy entering, leaving and accumulating within the system (i.e. within the tank). The energy balance equation may be expressed in words as

$$\begin{pmatrix}\text{Rate of energy}\\ \text{accumulation}\\ \text{within the tank}\end{pmatrix} = \begin{pmatrix}\text{Rate of energy}\\ \text{entering}\\ \text{the tank}\end{pmatrix} - \begin{pmatrix}\text{Rate of energy}\\ \text{leaving}\\ \text{the tank}\end{pmatrix} \tag{3}$$

Energy accumulating within the tank will cause the temperature of the water to rise. The magnitude of the temperature increase will depend upon the amount of water present and the water's specific heat capacity, C_P. The specific heat of a substance is the amount of energy required to raise the temperature of a unit amount of the substance by one degree. So,

$$\begin{pmatrix}\text{Rate of energy}\\ \text{accumulation}\\ \text{within the tank}\end{pmatrix} = m\ C_P\ \frac{\Delta T}{\Delta t} \tag{4}$$

Here m is the mass of the water and ΔT is the rise in temperature occurring over a time period of Δt.

Energy enters the tank in two ways: through the electrical heater and with the water flowing into the tank. Energy is associated with the flowing water because of its temperature and speed. Increasing with the water's temperature or speed will increase its energy. For almost all systems including the present one, the energy arising from the movement of the water will be negligible compared to the other energies in the system and so may be ignored.

The energy per unit mass associated with the temperature of the water is given by:

$$\begin{pmatrix}\text{Energy per}\\ \text{unit mass}\end{pmatrix} = C_P\ \left(T - T_{ref}\right) \tag{5}$$

where T_{ref} is some arbitrary reference temperature usually taken as 0ºC for water. We may therefore write an expression for the rate at which energy enters the tank:

$$\begin{pmatrix}\text{Rate of energy}\\ \text{entering}\\ \text{the tank}\end{pmatrix} = q + F\ C_P\ \left(T_i - T_{ref}\right) \tag{6}$$

In this equation, F is the mass flow rate of the water entering the system and T_i is the temperature of the inlet water.

If we assume that the tank is very well insulated and no heat escapes to the surroundings then we may write:

$$\begin{pmatrix}\text{Rate of energy}\\ \text{leaving}\\ \text{the tank}\end{pmatrix} = F\ C_P\ \left(T - T_{ref}\right) \tag{7}$$

In this equation we recall that because the contents of the tank are well mixed the temperature of the water leaving the tank is equal to the temperature throughout the tank.

Substituting expressions (2), (4), (6) and (7) into the energy balance equation (3) we find that:

$$m\ C_P\ \frac{\Delta T}{\Delta t} = K_c\ \left(T_s - T\right) + F\ C_P\left(T_i - T\right) \tag{8}$$

We then supply the teachers with some numbers for the variables above. We assume that the water in the tank is initially at 20ºC, and that the water enters the tank at 20ºC. The volume of the tank is assumed to be exactly 4 litres. If we take the density of water as 1.000 kg/L then the mass of water present in the tank will be 4.000 kg. The rate at which the water flows into and out of the tank is assumed to be 0.250 L/min, or 0.0150 L/s. The specific heat capacity of water varies with temperature, however we assume that over the temperature range of interest to us its value is constant. Finally we will assume that at time t = 0, the set point, T_s, is increased from 20ºC to 60ºC and then remains constant. These values are summarised in Table II.

TABLE II
SUMMARY OF THE SYSTEM CHARACTERISTICS

Initial temperature	T_i	20°C
Mass of water in tank	m	4.00 kg
Flow rate of water entering and leaving tank	F	0.0150 kg/s
Specific heat capacity of water	C_P	4.18×10^3 J/kg °C
Set point	T_s	20°C for $t < 0$
		60°C for $t \geq 0$

Immediately the set point is increased to 60ºC the controller begins supplying heat to the tank via the electric heater. As the water temperature in the tank begins to increase, the rate at which heat is supplied to the tank begins to ease off, as the power supplied to the heater is cut back. As time passes the rate at which the temperature increases, reduces as the temperature reaches an asymptote. Equation (8) may be used to predict the increase in the tank's water temperature as the controller responds to the step change in the set point temperature. If T_n is the water temperature in the tank at some time $t = n\ \Delta t$, and T_{n+1} is the corresponding temperature at $t = (n+1)\ \Delta t$ then by definition

$$T_{n+1} - T_n = \Delta T \qquad (9)$$

Substituting this into equation (8) and re-arranging we derive an expression which predicts the temperature in the tank after an increment in time based upon the temperature prior to the time increment:

$$T_{n+1} = T_n + \frac{\Delta t}{m\ C_P}\left[K_c\ \left(T_s - T_n\right) + F\ \ C_P\left(T_i - T_n\right)\right] \qquad (10)$$

The temperature following successive time steps may be calculated easily using a spreadsheet package such as Microsoft Excel. Figure 4 shows a sample of the Excel spreadsheet used to study the response of the temperature controller. The temperature at $t = 10$ seconds stored in cell E4 is calculated using equation (10). In terms of the spreadsheet cells this becomes

E4 = E3 + ($D4 - $D3) / ($B$7 * B9) * (E$2 * (B4 - E3) + B6 * B9 * (B5 - E3)) (11)

The formulae in the remaining cells in column E are generated by dragging the formula for E4 down the column.

	A	B	C	D	E
1				Time	Kc
2				(seconds)	40
3	K	80		0	20
4	Ts	60	deg C	10	20.957
5	Ti	20	deg C	20	21.855
6	F	0.015	kg/s	30	22.698
7	m	4	kg	40	23.489
8				50	24.232
9	Cp	4180	J/kg deg C	60	24.929
10				70	25.583
11				80	26.197
12				90	26.773
13				100	27.314
14				110	27.822
15				120	28.298
16				130	28.746
17				140	29.165
18				160	29.953
19				180	30.644
20				200	31.251
21				225	31.915
22				250	32.478
23				275	32.954
24				300	33.357

FIGURE 4
A SAMPLE OF THE SPREADSHEET USED TO STUDY THE RESPONSE OF THE CONTROLLER.

Figure 5 shows how the water temperature in the tank increases with time for four different values of K_c, the proportional gain. Several interesting results are revealed in Figure 5. We see that even if the system is left for infinite time the water temperature will never reach the set point of 60ºC. This is because after a long period of time energy no longer accumulates in the system as the amount of energy entering the tank through the heater exactly balances with the amount of energy leaving the tank as heat in the heated water.

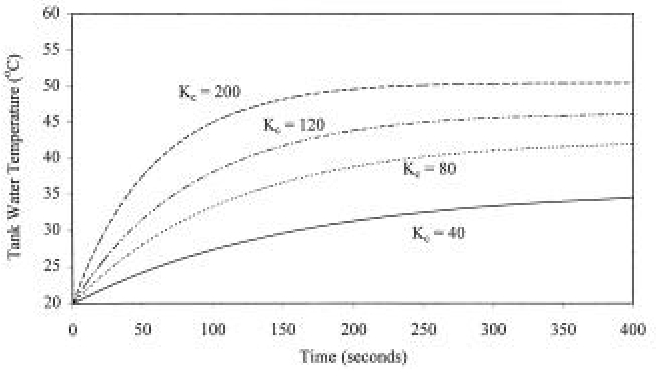

FIGURE 5
CHANGE IN THE WATER TEMPERATURE IN RESPONSE TO A STEP CHANGE IN THE SET POINT FOR FOUR DIFFERENT VALUES OF THE PROPORTIONAL GAIN.

Students are encouraged to explore the response of the system to variations in the system characteristics.

The challenge with this particular exercise is to present the material to the teachers in such a way that they both understand and are confident with it. The drawback of this example is that its contains many challenging concepts that will be new to teachers and students alike, and which must be clearly understood before any calculations may be performed.

CONCLUDING REMARKS

It is the author's experience that most secondary school mathematics teachers are very keen to provide their students with examples of real applications of mathematics. The vast majority of teachers, including the most skilled educators, do not have the background to develop the sorts of problems presented in this paper. The response of the teachers to these and other examples of engineering-based mathematics applications has been extremely positive. The use of such examples in the class room not own assists in keeping students interested in their mathematics studies but also acts to raise the profile of the engineering profession in the secondary school community.

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