IMPROVING STUDENTS' UNDERSTANDING OF ENGINEERING MATHEMATICS BY USING UNTRADITIONAL ASSESSMENT METHODS

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Abstract— *This paper describes how the computer algebra system (CAS) Maple 6 has been used by computer engineering students to learn mathematics, how the assessment method has been designed to be in harmony with the students' activities during the semester and how the assessment promotes learning. The paper also gives a rationale for teaching mathematics to computer engineering students.*

In order to avoid the use of Maple being treated as an appendix to the "important" part of the subject (i.e. the part being tested in the final examination) we have incorporated the use of CAS in the assessment method. This has been done by letting the students write a paper on one or several of the topics in the syllabus.

After having used this assessment method twice we can conclude that:

- *students prove that they are capable of writing a paper on mathematics relevant to the course they attend*
- *students express that the assessment method gives them the opportunity to achieve a deeper learning in the topic they have chosen*
- *while preparing the paper many of the students realise that the level they have chosen is too low. The result of this observation is that they expand their work to new, related and often more advanced fields. In this way the students choose to expand the syllabus in order to make good marks*
- *the students address the modelling aspect of mathematics to a far greater extent than earlier.*

MATHEMATICS IN NORWEGIAN ENGINEERING EDUCATION

The Norwegian engineering education is defined by a *frame curriculum*, that is a national curriculum giving directions for the content and level of the educational programs in engineering given by the different colleges. The education takes 3 years to complete, and gives 180 credits.

The frame curriculum was developed by the Engineering Education Council during the years 1994-1995 and was given its final approval by the Ministry of Education in 1996 [1]. During the developing period it was stressed that the content and level of the education should be comparable to the corresponding educational programs given in the other European countries. That made it necessary to base the frame curriculum on science and mathematics.

The curriculum in mathematics is the same for all the different branches of engineering (computer science, chemistry, civil engineering, electrical engineering and so on.) There are three different courses in analysis, a course in linear algebra and discrete mathematics and finally a course in statistics and probability theory. The five courses give a total of 30 credits, 24 of which are compulsory and 6 can be elected.

According to the frame curriculum the instruction in mathematics shall put emphasis on mathematical modelling and strategies for solving problems. In addition all the courses shall give the students experience in using computational software when solving problems.

The course we are discussing in this paper is one of the three courses in analysis and is given to computer science students at *The Norwegian School of Information Technology* (NITH). It covers topics such as functions of multiple variables, Laplace transforms, Taylor- and Fourier series.

A RATIONALE FOR TEACHING MATHEMATICS TO COMPUTER SCIENCE STUDENTS

The frame curriculum [1] states that the mathematics courses shall

- ensure that the students have a theoretical platform and necessary tools for further studies in engineering courses
- give the students a foundation for further studies
- give the students a “language” for effective communication in a technological/scientific community
- ensure that the students shall be able to read literature based on mathematical skills.

These are broad and widely accepted goals for any technical and scientific education and few argue against them. However, the goals give little concrete information on how to teach and what to teach. In a way they are to be treated as a basis for the development of study plans and curricula.

It is mandatory for all Norwegian engineering colleges to follow these goals. But the “have to” aspect rises several interesting questions when *the same* courses must be taught to *all* kinds of engineering students: Are all the topics relevant to all students? Are all the topics relevant to every practising engineer? Should there not be different syllabi for civil engineering students and computer science students?

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These are questions that have to be answered with great care because they touch the very core of the arguments for teaching mathematics to engineering students. In my opinion every teacher (not only in mathematics, but in *every* subject) should be able to give strong arguments for why *their* subject defends a place in an educational program.

Driver et. al. [2] and Sjøberg [3] construct categories for why science should be taught to all students. These categories (according to Sjøberg) are:

- The utility argument
- The economic argument
- The democratic argument
- The cultural argument

Even though these categories have been made up to defend science's place in our schools, they apply equally well to mathematics in engineering education. Let us look further into these arguments, and with the computer science students in mind.

The Utility Argument

The utility argument is the most frequently used when arguing for a subject or for special topics in a given subject. All the general goals in the frame curriculum (see above) can be placed in this category. The authors of the frame curriculum have obviously meant to give the students tools to cope with the problems and challenges they will meet when studying computer science subjects and when working as professionals. There seem to be no intention of making the study of mathematics a goal in it self. That is left to those who aim at academic studies at the universities.

But the utility argument has to be somewhat refined. We have to ask ourselves: To whom shall mathematics be useful? And: When shall the subject be useful to those who have studied it?

One answer to the first question is obviously *to the student*. But the answer can also be: to the employers of engineers and the information technology industry. Many branches of the industry are based on applying computers to solve technical calculations and to those it is important with a sound background in mathematics.

The second question is well answered by the general goals: Mathematics shall be useful both when studying in engineering colleges and when working as professionals. It is of course motivating to the students when they see the usefulness of mathematics, and every effort should be made by the teaching staff to make this clear.

The Economical Argument

For the industry it is important to have a staff that can cope with new challenges without having to invest too much in further education. Mathematics is usually treated as a basic subject and the rate of change in content is low compared to computer science subjects. It should therefore not be the primary goal to invest in further education in mathematics for engineers, but rather take it for granted that they are familiar with the traditional topics.

But the economic argument also applies to each individual. To use time and resources learning basic mathematics should not be the priority for skilled engineers. This justifies the place of mathematics in engineering education.

The Democratic Argument

Most engineers work in teams where the members have different spheres of competencies and different areas of expertise. It is of great importance to every one to be able to attend discussions on equal footing. As mathematics is a common subject to all engineers, a lack of insight can result in being left out of democratic processes concerning daily work, strategic decisions and reorganizations.

The Cultural Argument

During the second half of the 19th century the engineering profession was institutionalized and founded on a base of science and mathematics [4]. Since then engineering education has been given by specialized colleges and universities, and there have been engineering societies taking care of the common heritage of the profession. During the 20th century the interaction between science, mathematics and technology has increased and is today best viewed as a "seamless web" [5]. It would be very wrong if computer science engineers did not enter this common knowledge base.

None of the four categories of arguments given above has any obvious first rank. When planning computer science education one has to take each and one of them into account, and when arguing for mathematics' place one has to be aware of the existence of the four positions one can take.

ADVANTAGES OF USING COMPUTER ALGEBRA SYSTEMS

Having established a rationale for teaching mathematics to computer science students, one has to decide which topics shall be covered, how the teaching shall be organized and how learning shall be enhanced. I shall not elaborate on the work that has to be done when selecting the topics in mathematics but instead focus on how a Computer Algebra System (CAS) can help increasing students understanding of mathematics.

Since the early 1960's research works on constructing algorithms and systems for performing symbolic mathematics have been under way, for example at MIT. In November 1980 *the Maple project* started out with the aim to create a portable system not demanding a huge amount of

RAMs and extensive CPU-time [6]. It became clear that Maple had a commercial market, and in 1987 Waterloo Maple Software (now Waterloo Maple Inc) was founded. The Maple project has not terminated and still new features are added to the system. (There exist a range of different CAS, such as Mathematica and Derive, but as we have used Maple in our school I shall not comment on these.)

One of the intentions of CAS is the "mechanization of mathematics", which means to make laborious calculations by using calculator, paper and pencil unnecessary. In an educational situation this will give the students the possibility to focus on understanding principles rather than performing tedious work. However, this raises the important question on what students have to learn by doing mathematics "by hand" and what can be skipped in favour of CAS.

In the setting of computer science education we have to take into account that the students in Norway have completed 12 years of mathematical education before entering college. This means that they already possess basic skills and the discussion of leaving the "handwork" can be made a lot easier. In the case I describe here the students also have passed courses in engineering mathematics equivalent to 12 credits work.

Let me therefore just briefly mention some of the advantages of using CAS:

- Standard calculations, both numerical and algebraic, are performed at great speed and without error
- It is possible to plot graphs of functions both in two and three dimensions. Especially the feature of plotting three-dimensional graphs saves a lot of time and makes it possible to get a good visual understanding of the geometry of the surfaces.
- It is possible to perform calculations of much higher complexity than "by hand". This opens the opportunity to work with more realistic examples than when not using CAS.
- Due to the high speed in which the calculations are performed, it is easy to experiment. To investigate what happens when the value of a parameter is changed is most often not a big deal.
- The time saved by letting the CAS perform the calculations can be used to place focus on principles rather than technicalities.

In addition to the advantages of using CAS in mathematics The Norwegian School of Information Technology has the intention of helping students to become "computer literate citizens". Even though our students are becoming specialists in computer science it is a goal to give them experience in using computers in every subject they study. Mathematics is not a subject to be left out in this content.

To conclude, we can say that using CAS has as its aim to improve the understanding of mathematics and give a contribution to the students' computer literacy.

ORGANIZING THE TEACHING AND ASSESSMENT WHEN USING A COMPUTER ALGEBRA SYSTEM

One of the imperatives in the engineering education at NITH is to prepare the students for their work as professional engineers. Perhaps the most important ability in this regard is to work together with other professionals in a team. When introducing Maple as the main tool for doing mathematics, the idea occurred that it would give extra learning outcome if the students' work was organized in groups. In this way they could approach new mathematical topics under conditions comparable to a real working situation.

We decided that the use of paper and pencil should be banned during the course and make the students rely completely on the use of Maple. This had, however, the consequence that we had to rework the way we assessed the students.

Since the students should use Maple throughout the whole course, we found it meaningless to test them with a traditional written individual exam. Therefore we decided that the students should submit a paper as their final exam.

Let us look in further details at how the course was organized, purposes of assessment and how student assessment was performed in practice.

Introducing the Students to Maple

There was not given any particular instruction or lectures on how to use Maple. As the students' ability to use Maple was to be assessed we found that it was most convenient to let them try out the different features in the program by themselves. Being computer science students it was also relevant to let them try out new software by themselves. The Maple Help-function is a rich source for finding out how the program works, and the Waterloo Maple web-site [7] gives lots of relevant examples.

However, for each week during the term, the lecturer published Maple worksheets introducing new topics in mathematics. By studying these worksheets the students were able to see how relevant Maple-sequences were to be constructed to solve relevant mathematical problems.

Lectures and Group Work

Each week for a period of 10 weeks there were given 2 or 3 hours of lectures in a lecturing theater. All lectures were given by introducing the new topics with the aid of Maple. In addition to the lectures there were 2 hours of guided group work in the computer laboratory.

In the first lecture all the different main topics were introduced: *Functions of several variable*, *sequences* and *series*, and *Laplace transforms*. Each of these three topics was then studied for two weeks. In the last four weeks each topic was elaborated upon, and special practical uses were studied.

Relevant exercises were distributed through the school's Intranet. Many of these exercises were constructed in such a way that the students could modify similar examples already distributed to them to solve the actual problem.

Purposes of Student Assessment

The assessment of the students' performance has many purposes. I shall not go into details here, but let me briefly mention some of the most important aspects. For further information [8] gives good information.

Perhaps the most important purpose of assessment is to enhance learning. To give relevant feedback as a support in the learning process is of great importance. The feedback should be given in such a way that the student can act on it and improve weaknesses recognized by the test. This kind of assessment is called *formative*.

Summative assessment is used for certification of the student's abilities and gives an overview of previous learning. It should be obtained by accumulation of test results over time.

It is also important to remember that student assessment is an important part of showing to the public how well a school performs. Be accumulation of students' test results it is possible to compare different schools when the tests are the same. To use such data to rank schools is controversial because the students' backgrounds are not taken into account. In this way assessment is used for *accountability*.

How the Assessment Was Performed

The main purpose of the assessment in the course we describe here is to improve students' learning. The goals are that the students shall:

- understand the mathematical topics presented in the course
- be able to define a problem relevant to the curriculum
- be able to use Maple to solve the problem defined by the student
- write a paper that presents the problem, discusses theory, presents the solution and discusses the results
- be able to perform the tasks above by working together with other students in a project group.

During the first year of study the students' abilities in mathematics have been tested in an individual written exam. When we introduce group work in the assessment in mathematics in the second year, there is always a risk that some of the group members will not work as hard as they should and try to "surf along". The first year exam, however, will account for each individual's skill in mathematics.

In the first lecture the students were given an orientation of the assessment in the course. During the first "survey lecture" they were motivated to start thinking about which of the three main topics they found most interesting. Then, during the two-week periods of work with the different topics, the students were given further inspiration to try to find interesting ways to approach the writing of the paper.

At about midterm it was given a lecture on how to write a scientific paper. Then the students had to write a concrete project proposal where they made it clear what problem they wanted to address and how they were to solve it. The project specification had to be approved by the lecturer before the students could start their work. Not all the groups' proposals were approved at once, but through iterative processes all groups had worked out relevant specifications.

For the last four weeks of the term the students worked on their project along with attending lectures and group work. The groups could contact the lecturer to receive guidance in their work.

The papers were submitted for assessment at the end of the term, and were assessed by the lecturer and an external examiner.

THE PROJECTS

When selecting projects the students have chosen themes from all the three main topics in the course. During the two years we have used this kind of assessment, the distribution between the topics is about equal.

The students have shown great creativity in how to approach the work. Many of the groups have tried to find applications for the basic mathematical themes they have learned during the course. On the other hand, some of them have chosen to write a paper in "pure" mathematics, often with the aim to help other students to understand the theme.

Some of the project titles are:

- How to use Laplace transforms to model cruise controls in cars
- Control theory and Laplace transforms
- A mathematical model of a filter used to split ADSL- and ISDN-signals
- Laplace transforms and FIR- and IIR-filters
- Modeling of economic transactions using functions of several variables
- Finding the shortest way between two places in a modeled landscape
- The use of functions of several variables in physics and chemistry
- Simulation of high frequent signals using Fourier series

The titles show that the students choose to write papers on themes not usually found in standard courses in engineering mathematics, except for short examples. The papers have an adequate balance between basic theory and application of the mathematical principles, and most of them follow established standards for the lay-out of a scientific paper, including references to other papers, text books and web-sites. In this way the students make it clear that they are capable of combining theory and practice and document it in a professional way.

With respect to students' achievements all the groups have passed the exam so far, and with fairly good marks.

As lecturer I find it very satisfying when I am told from students that "I didn't know that mathematics could be fun!"

CONCLUSIONS

After having used this assessment method twice we can draw the following conclusions:

- The students prove that they are capable of writing a paper on mathematics relevant to the course they attend.
- The students express that the assessment method gives them the opportunity to achieve a deeper learning in the topic they have chosen.
- While preparing the paper many of the students realize that the level they have chosen is too low. The result of this is that they expand their work to new, related and often more advanced fields.
- The period of formulating the problem to be solved is extremely valuable because the students have to find out what problems are relevant. This makes it necessary to understand the mathematical principles.
- By choosing to focus on applications the students expand the syllabus in order to make good marks.
- The students address the modelling aspect of mathematics to a far greater extent than earlier.
- The students are capable of learning how to use Maple without specific instructions
- It is possible to use the papers to mark the students' performance. The correlation between the two examiners' markings is as high as it is on traditional exams in mathematics.
- By organizing the assessment as a group-based project the students get experience in how to approach a new field of study in a "real-world" setting.

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