A NOVEL Approach to PID Controlled Design

Fred AWAD[1]

1. Department of Electrical Engineering, Ecole de technologie superieure, Montreal Qc. Canada

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ABSTRACT

The purpose of this paper is to present a new technique for pole-placement using PID controllers. It is based on solving two simultaneous nonlinear equations. The development of the equations, as well as their solution, is performed on a programmable hand-held calculator.

In solving the two simultaneous equations, the numerical values of two parameters can be determined. For example the value of the gain k and the magnitude of a complex pole can be the two unknown variables to be calculated. Alternatively the locations of the two real zeros of the transfer function of the controller can be obtained if the proper conditions are imposed.

The advantages of this technique include a higher precision compared to the graphical techniques described in most text books and a better compatibility with modern engineering education approach where numerical calculations have often replaced graphical solutions. In addition, this technique allows students to obtain fast results in a classroom environment and accurate answers in tests and examinations. Portable PC's are not required and no programming is needed.

I – INTRODUCTION

The PID controller is the most popular form of linear control algorithms (1) that are used in process control. It is estimated that 95% of processes can be properly controlled using such a controller (2). It is not surprising then that text books intended for undergraduate courses in control theory (3, 4) tend to consecrate important portion of their text to analyze system performance using different PID controllers and to explain how to design the parameters of such controller to meet some defined specifications.

The most common techniques used for designing controller's parameters take place in either the frequency domain or the time domain. Designing in the frequency domain uses either a Nyquist or Bode diagram and tends to modify such a diagram to increase the phase margin and / or the gain margin. Time domain analysis on the other hand uses the root-locus technique to modify the locations of the system poles. In other words it is a form of pole placement algorithm. In this paper we will limit ourselves to the time-domain analysis.

The root-locus technique is a graphical approach that plots the possible locations of closed-loop systems. To determine the points that are part of the root-locus graph using a digital computer is a number-crunching exercise that requires a powerful CPU. The motivation for this paper is our search for a calcutor-based solution to the PID design problem. This allows students to do the necessary calculations either in the clan room or during major examinations. At our school, students are required to have at their disposal a Texas Instruments Voyage 200 hand-help programmable calculator they use in several course.

The structure of this short paper is as follows: in section 2 we present the mathematical foundations of the proposed technique. Some examples follow in section 3 to illustrate the power of our proposed approach. In section 4 we present our conclusions.

[1] E-mail: Fred.Awad@etsmtl.ca

II – THE TECHNIQUE

The technique that we propose takes advantage of the fact that an equation of the complex variable s is equivalent to two simultaneous real-variable equations that, when solved, would give the complex value of s. Notice that with our calculator; the conversion from complex variable to real- variables is done by the calculator. As well the solution of the two nonlinear equations that come out of this conversion.

The characteristic equation of a linear time-invariant system can be written in the form:

$$F(s,k) = 0 \qquad (1)$$

Where s is the Laplace variable and K is the forward system gain. This equation is complex. By replacing s by its equivalent rectangular complex form σ + jω the characteristic equation becomes

$$f(s,k) \mid_{s=\sigma+i\omega} = f(\sigma + i\omega, k) = g(\sigma, \omega, k) + i\, h(\sigma, \omega, k) \qquad (2)$$

Where *i* is the imaginary operator

To find the solution of the complex equation (1) is equivalent to solving the two simultaneous equations

$$g(\sigma, \omega, k) = 0 \qquad (3)$$

and

$$h(\sigma, \omega, k) = 0 \qquad (4)$$

Since we have only two equations, only two unknown parameters can be calculated. For example the values of σ and w can be determined assuming the value of k is known.

An alternate approach which can be more convenient in certain situations is to use the polar representation for s i-e.

$$s = \angle\theta \qquad (5)$$

This substitution is more useful when the angle θ is specified through the damping factor or the percentage overshoot.

III – NUMERICAL EXAMPLES

Example 1:

Consider the system described by the block diagram of figure 1. It is required to calculate the value of the gain k as well as the response time Ts when the closed loop response has a 4 % overshoot.

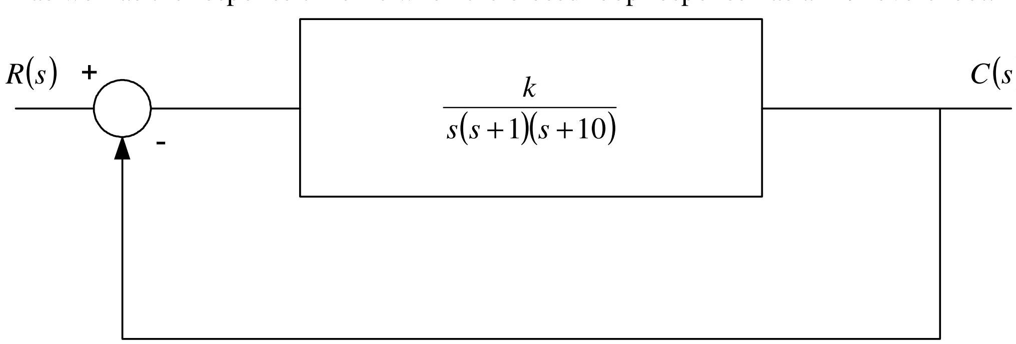

Figure 1: block diagram of a control system.

[1] E-mail: Fred.Awad@etsmtl.ca

Solution:

Since the overshoot is 4 %, the dominant poles have a damping factor of 0.707. We can therefore write $s = r\angle 135^\circ$. The characteristic equation of the closed loop system is

$$s(s+1)\,(s+10) + k = 0 \quad (6)$$

Substituting $s = r\angle 135^\circ$ in equation (6) we get (7)

$$0.707r^3 - 7.07r + k + 0.707r\,(r^2 - 15.55\ r + 10)\,i = 0$$

Equation (7) is equivalent to the following two real polynomial equations in r and k.

$$0.707r^3 - 0.707r + k = 0 \quad (8)$$

and

$$r\,(r^2 - 15.55\ r + 10) = 0 \quad (9)$$

The appropriate solutions of these two equations are

$r = 0.672$ and $k = 4.536$

The third root is located at $s_3 = -10.05$.

The complex roots are therefore dominant and they determine the response time. They can be expressed in rectangular form as

$$s_{1,2} = -0.475 \pm 0.465i \quad (10)$$

The time constant $\tau = 1/0.475 = 2.105$ and the response time $T_s = 5\tau = 10.526$s.

Example II

Consider the system described by the block diagram of figure 2. It is required to calculate k and z so that the complex poles are located at $s = -4 \pm i3$

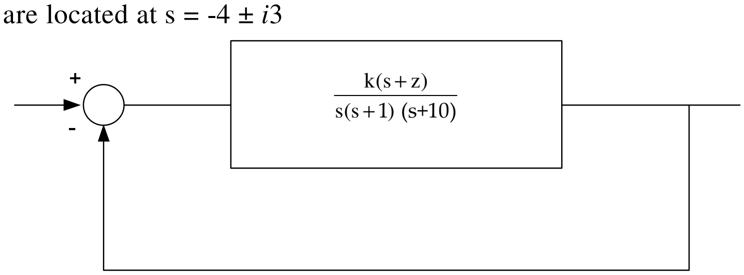

Figure 2

Solution:

The characteristic equation of this system is

$$s(s+1)\,(s+10) + k\,(s+z) = 0 \quad (11)$$

Since $s = -4 + i3$ should be a solution; we replace this expression in equation (11) to get

$$Kz - 4k + 81 + (3k - 117)\,i = 0$$

[1] E-mail: Fred.Awad@etsmtl.ca

Whose solution is: k = 39 and z = 1.923

IV- CONCLUSION

We have presented a simple technique for calculations the parameters of a PID controller using a hand-held calculator. The technique in conceptually simple and the calculations for system up the order 4 take a few seconds. This technique is well adapted for solving exam problems where computer power is normally not available.

A part from PID calculations the same technique can be used to solve multitude of control problems where the value of some parameters are to be calculated.

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[1] E-mail: Fred.Awad@etsmtl.ca