Investigative Projects in Engineering for Secondary School Students 2

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Abstract — *Engineering faculties around the world must always work to ensure that they attract high standard students to their courses. Unfortunately prospective students have a poor appreciation of the engineering profession. They are often unaware of the diversity of challenges and opportunities that await them in an engineering career. A number of strategies exist to raise the profile of engineering within secondary schools. One activity involves targeting the students while still at school, with engineering academics either visiting the schools or playing host to the students on campus. An alternative to this is to present to mathematics teachers real engineering design problems that may be solved by the application of relatively simple mathematical concepts. This paper describes two activities for use in the senior secondary mathematics class room which have been developed to illustrate how mathematics is used by engineers. The storage of bulk solids such as coal, gravel, iron ore and wood chips provide the background for the first activity. A series of simple experiments have been developed which allow the formation of solid heaps to be studied in the class room. Using everyday solid materials of a range of properties such as sand, rice, milk powder, sugar and confectionary students are able to take a range of measurements relating to the shapes of the heaps. Using simple geometry they can then characterize the shapes of the heaps. Mathematics topics covered include statistics and geometry while engineering topics include the storage of bulk solids, the angle of repose of the heaps and the sphericity of the solid particles. The second activity relates to the characterization of oil fields. After a brief introduction which includes a description of underground petroleum reservoirs and how oil wells are drilled students are given graphical data relating to the physical properties of an actual petroleum reservoir. This data relates to the thickness of the field as well as the field porosity and oil saturation. Mathematics topics addressed by this topic include graphing and averaging while the context gives the students an introduction to petroleum engineering.*

Index Terms — *Outreach, secondary school, mathematics.*

INTRODUCTION

Over the past decade the author has been working with secondary school mathematics teachers to develop material for the school class room that illustrates the role that mathematics plays in engineering. As well as hopefully making the teaching of mathematics more interesting by giving it a context, this activity helps to raise the profile of the engineering profession within the secondary school community. To many students and even their teachers, the popular image of the engineer is a man who wears a hard hat and builds bridges and tall buildings. Unless the student has a close relative who is themselves an engineer, the students will often have no understanding of the diverse range of activities that engineers engage in. Incorporating engineering-themed material into the class room can only benefit both the engineering profession and the teaching of mathematics.

Any material developed for the secondary school class room must meet a number of criteria if it is to be successfully deployed:

- The level of mathematics covered must be appropriate. Some otherwise excellent examples of engineering applications of mathematics have been developed which call on mathematics skills beyond the ability of the target students and in some cases even their teachers.
- The context in which the example is set must not require too much background information to be understood by the student. Teachers must be confident that they completely understand the context in which the mathematics is set.
- The context must be interesting to the age group of the students. Any topic which is related to the environment will usually be of interest.
- The context should be one that will be equally of interest to both males and females.

The author has developed material over the past decade which presents examples of how simple mathematics is used by engineers. The topics covered include:

- The design of a bulk liquid storage facility using volumes of cylinders and rectangular prisms [1,2].
- Pasteurisation of cream. This context covers simple curve-fitting [3].

- Controlling processes. Proportional and proportional-integral control is considered in an activity ideally-suited to a spreadsheet calculation [4].
- Underground car park structure design. The design of an innovative carpark in Melbourne is considered in which students must specify the shape of the columns in Cartesian and parametric form [4].
- Estimation of the capital cost of a chemical processing plant by the use of simple ratios [5].
- The geometry involved in the design of interlinking blocks in a 19th century stone lighthouse [6].

In this paper two further examples of material for the secondary school class room are presented.

BULK STORAGE OF SOLID MATERIALS IN HEAPS

The storage of bulk solids such as coal, gravel, iron ore and wood chips in heaps provides the background to a simple investigative project in mathematics. Mathematics topics covered include basic geometry volumes of cones and other axisymmetric shapes as well as basic statistics. A simple experimental program is developed which students can either undertake in the class room or at home. This program investigates the angle of repose of the heaps formed using a range of different solid materials. Rather than use coal, gravel, iron ore or wood chips which would be impractical instead students investigate the shape of heaps formed from materials including sand, rice, milk powder, sugar and a type of sugar confection known in Australia as 100's and 1000's.

The key to this investigative project is the simple nature of the experimental program. The experiments use items found in most kitchens of the Western World. The equipment required consists of a single kitchen funnel, two set-squares and a ruler. Also required is a sheet of paper with concentric circles of known diameters bisected by two lines at right angles as shown in Figure 1. A retort stand and clamp may also be required but these can be replaced by the more inventive students using children's construction kits such as Lego.

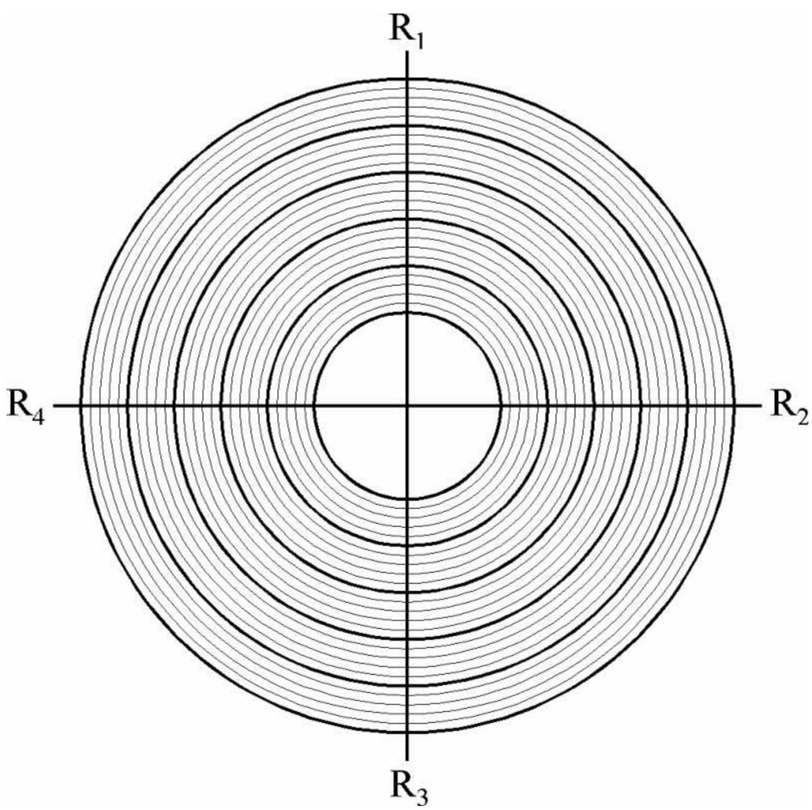

FIGURE 1
TARGET SHEET OF CONCENTRIC CIRCLES.

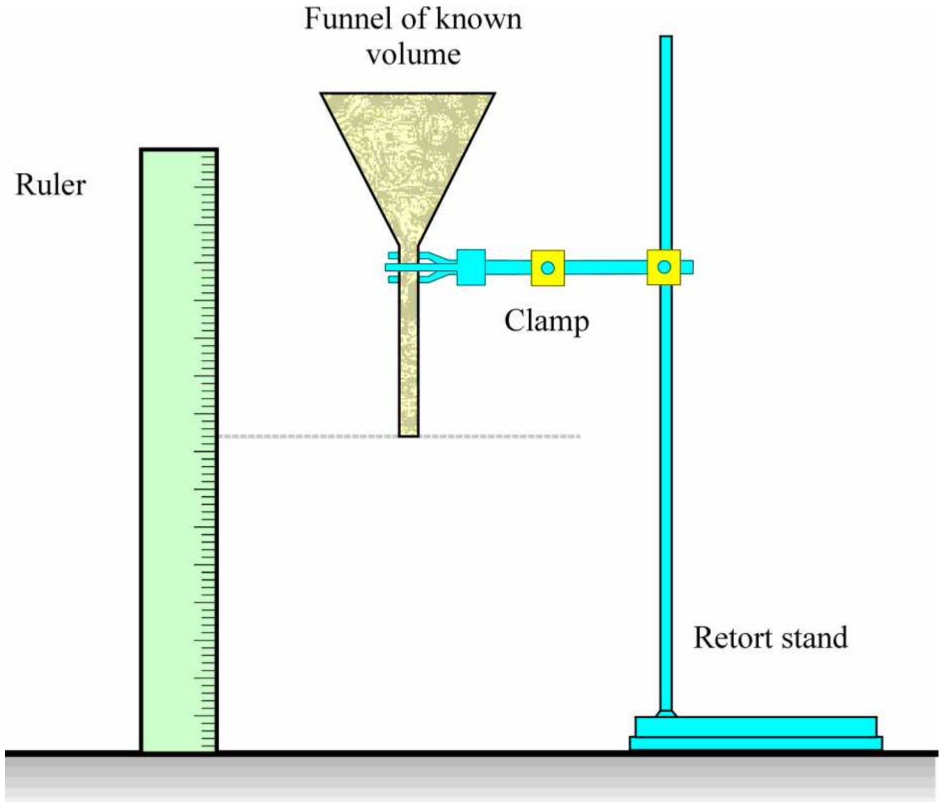

FIGURE 2
THE FUNNEL IS POSITIONED DIRECTLY OVER THE TARGET SHEET AND FILLED WITH THE SOLID.

The students begin by accurately determining the capacity of the funnel. This in itself can be a challenge. Some students might choose to accurately measure the funnel's dimensions and then use those measurements to determine the volume of the conical and cylindrical sections of the funnel. Other students might chose to fill the funnel with water before allowing the water to drain into a vessel resting on the balance. Knowing the mass and density of the water they can then determine the capacity of the funnel.

The students then place a sheet of paper upon which a series of concentric circles of known diameters have been drawn. Two bisecting lines at right angles are also drawn on the paper (Figure 1). This sheet is the target and the circles will be used later to measure the diameter of the base of the heaps.

Using the retort stand and clamp the funnel is held vertically directly over the centre of the target. Its position is adjusted so that the open end of the funnel is a fixed height directly above the target (Figure 2). Depending on the capacity of the funnel the height may be 10 cm, 15 cm or 20 cm. Holding a finger over its open end the funnel is then completely filled with the solid material so that the level of the solid within the funnel is level with the top of the funnel. The finger is removed and the solid allowed to flow out of the funnel, dropping down onto to target sheet and forming the heap.

The task of the students is now to measure the key dimensions of the heap. Milk powder forms an almost perfect conical shape with only the very top being rounded. This is an ideal material to start the experimental program with. The height of the heap is measured using two set squares as shown in Figure 3. For each heap this should be done four times. Usually it is possible to measure the height to within about 1 mm. The diameter of the base of the heap is then measured using the set of concentric circles drawn on the target sheet. The positions where the heap base cuts the four radials, R_1, R_2, R_3 and R_4 are noted and used to calculate the average radius.

Using data for the average height and average base radius students can then calculate the heap's average angle of repose and volume.

Table 1 presents data sets for 10 different heaps each formed in exactly the same way. Students could be asked to investigate the effect of varying the height of the funnel outlet above the target sheet on the heap's angle of repose.

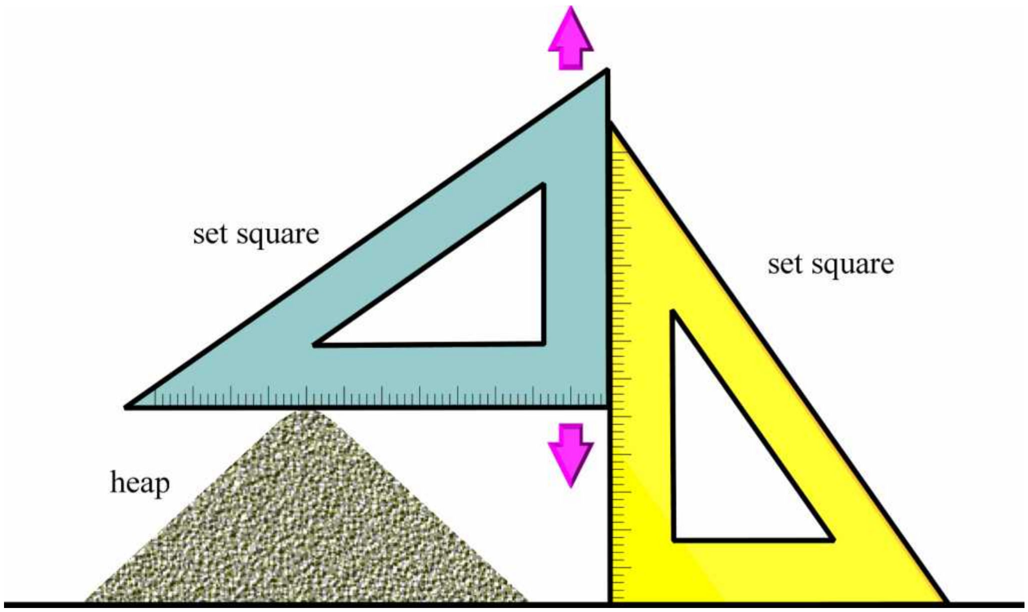

FIGURE 3
MEASURING THE HEIGHT OF THE HEAP USING TWO SET SQUARES.

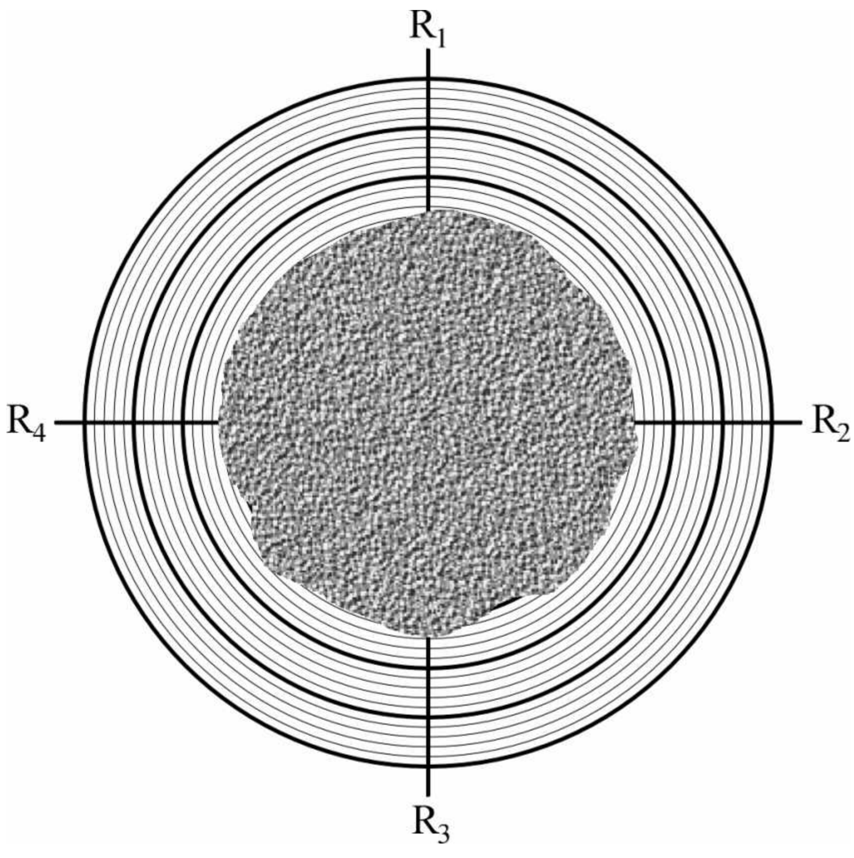

FIGURE 4
THE BASE OF THE HEAP IS MEASURED USING THE TARGET SHEET OF CONCENTRIC CIRCLES.

TABLE 1
EXPERIMENTAL RESULTS FOR THE SHAPE OF TEN HEAPS OF MILK POWDER. THE SHAPE OF THE HEAP IS THAT OF A SYMMETRIC CONE.

Height (cm)					Radius (cm)					Angle
H_1	H_2	H_3	H_4	Average	R_1	R_2	R_3	R_4	Average	
11.0	11.0	11.0	10.9	10.98	8.3	8.5	8.5	8.7	8.50	52.2
11.3	11.1	11.1	11.1	11.15	8.4	9.3	9.4	8.8	8.98	51.2
10.8	10.7	10.7	10.6	10.70	8.7	9.4	9.3	9.1	9.13	49.5
10.8	10.8	10.7	10.6	10.73	9.1	9.8	8.8	8.7	9.10	49.7
10.2	10.1	10.1	10.0	10.10	9.5	9.4	9.5	8.5	9.23	47.6
11.2	11.2	11.2	11.2	11.20	9.6	9.5	8.7	10.0	9.45	49.8
10.8	10.7	10.7	10.6	10.70	9.6	10.0	9.0	8.9	9.38	48.8
11.0	10.8	10.8	10.7	10.83	9.0	9.3	9.3	9.3	9.23	49.6
10.7	10.7	10.6	10.6	10.65	9.0	9.0	9.4	9.3	9.18	49.3
10.7	10.7	10.5	10.5	10.60	9.7	9.5	8.8	9.2	9.30	48.7

TABLE 2 : EXPERIMENTAL RESULTS FOR THE SHAPE OF TEN HEAPS OF RICE. THE SHAPE OF THE HEAP IS THAT OF A TRUNCATED SYMMETRIC CONE.

Height (cm)					Base Radius (cm)					Top Radius (cm)					Angle
H_1	H_2	H_3	H_4	Ave.	$R_{B,1}$	$R_{B,2}$	$R_{B,3}$	$R_{B,4}$	Ave.	$R_{T,1}$	$R_{T,2}$	$R_{T,3}$	$R_{T,4}$	Ave.	
3.5	3.4	3.4	3.4	3.43	7.2	7.2	6.5	6.9	6.95	1.25	1.00	1.25	1.00	1.13	30.5
3.5	3.5	3.5	3.5	3.50	7.5	7.2	7.5	6.8	7.25	1.50	1.25	1.50	1.25	1.38	30.8
3.5	3.5	3.4	3.4	3.45	7.4	7.4	6.6	6.7	7.03	1.00	1.50	1.25	1.25	1.25	30.9
3.5	3.4	3.4	3.4	3.43	7.6	7.4	6.5	6.9	7.10	1.25	1.00	1.25	1.25	1.19	30.1
3.4	3.5	3.4	3.4	3.43	7.0	7.4	6.9	7.1	7.10	1.50	1.25	1.50	1.25	1.38	30.9
3.4	3.5	3.5	3.5	3.48	7.5	7.1	6.8	6.8	7.05	1.50	1.25	1.25	1.25	1.31	31.2
3.2	3.4	3.3	3.3	3.30	7.0	7.0	7.0	6.6	6.90	1.25	1.25	1.50	1.50	1.38	30.8
3.3	3.4	3.4	3.4	3.38	7.2	6.7	7.1	7.4	7.10	1.25	1.50	1.50	1.25	1.38	30.5
3.4	3.5	3.5	3.4	3.45	7.2	7.1	6.8	7.0	7.03	1.25	1.25	1.25	1.50	1.31	31.1
3.5	3.5	3.5	3.5	3.50	7.3	6.8	7.0	7.0	7.03	1.25	1.50	1.25	1.50	1.38	31.8

Table 2 presents the data sets for 10 different heaps of rice. Rice forms a truncated cone having a flat horizontal surface at its top. Students could be asked to investigate the effect of varying the height of the funnel outlet on both the heap's angle of repose and the ratio of the heaps base and top radii.

This investigative activity allows students to explore several mathematical concepts against the engineering background of the storage of bulk solids. When first developed and presented to teachers in 1999, the teachers were given sufficient background material to allow them to develop the context in class.

RESERVOIR ENGINEERING

Like most of the rest of the wider community secondary school students have a very poor understanding of where oil comes from. They will generally know that it comes from underground often under high pressure but beyond these basic concepts their knowledge is scant. For many students oil comes from vast subterranean caverns which are filled with oil. Once a hole is drilled into the cavern the oil will flow to the surface until the cavern is empty. The following activity is aimed at correcting this myth and at introducing teachers and their students to some of the activities that petroleum engineers perform. This activity was presented to a conference of mathematics teachers in 2000.

The activity begins by describing what petroleum is and how it was formed. Petroleum reservoirs are then described. Petroleum is not found in vast caverns but within the pores of porous rocks. The engineering challenges of drilling, casing and completing a well is then described. The purpose of this material is to give the teachers some background information that they will find useful when the applying the material in the class room.

The following activities were developed to enhance students graphical skills.

1. Figure 1 shows the cross-section of an oil well before it is cased. Some of the rock on the well walls has caved in. Estimate the cross-sectional area of the well.

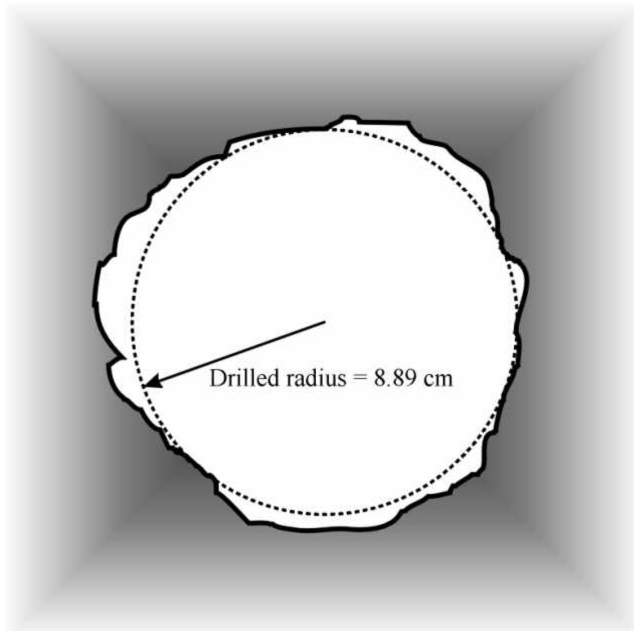

FIGURE 5
CROSS-SECTION OF AN UNCASED DRILLED WELL.

2. The thickness of the oil-bearing layers of a petroleum reservoir may vary considerably. It will usually be thinnest at the edges and thickest towards the middle. Below is a contour map showing the thickness of an oil reservoir. Estimate the volume of the oil-bearing formation. Express your answer in cubic metres.

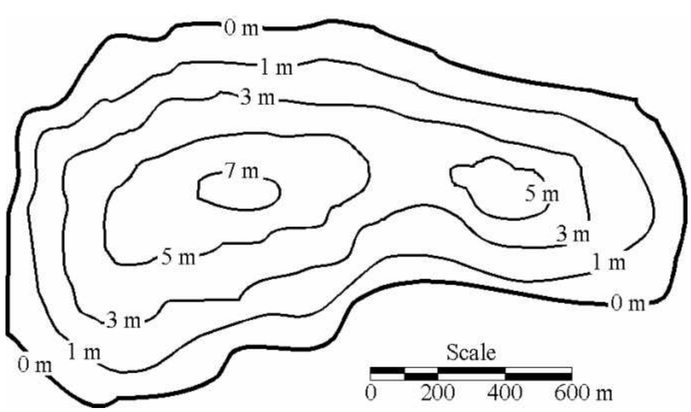

FIGURE 6
CONTOUR MAP OF THE OIL-BEARING FORMATION SHOWING THE THICKNESS OF THE FORMATION.

3. The underground layers of rock in which the oil formed millions of years ago are rarely horizontal. They are often tilted or even folded as a result of the movements in the Earth's crust of millions of years. Below is a map for the reservoir featured above which shows the distance between ground level and the top of the oil-bearing formation. How far below the surface is the deepest part of the reservoir ? Where is this point on the map ? Plot a graph showing the depth of the top and bottom surfaces of the oil reservoir along the cross-section A-B.

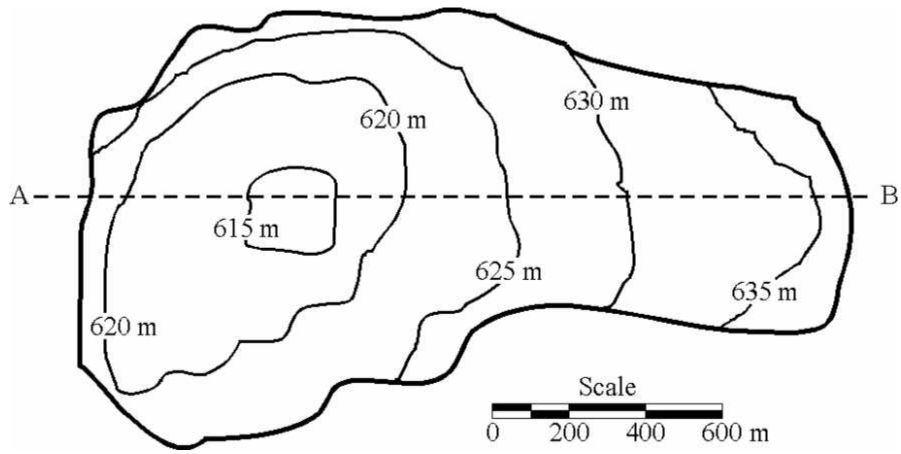

FIGURE 7
CONTOUR MAP OF THE OIL-BEARING FORMATION SHOWING THE DEPTH TO THE TOP OF THE FORMATION.

4. The oil within the oil-bearing formation is contained within the pores of the rock. The fraction of the total volume of a sample of rock that are open pores is known as the porosity. Some rock will be very porous and have a high porosity while other rock will be very tight and have a low porosity. The higher the porosity the better for oil exploration as this usually means there is a greater pore volume available to contain the oil. The porosity of a typical oil reservoir may range from around 10 % to 35 %. A sample of rock with a volume of 100.0 cm^3 and a porosity of 22.0 % will have a total pore volume of 22.0 cm^3 while the remaining 78.0 cm^3 will be the actual solid rock. The diagram below shows the variation of porosity throughout our reservoir. What is the average porosity of the reservoir? We will assume that porosity does not vary vertically.

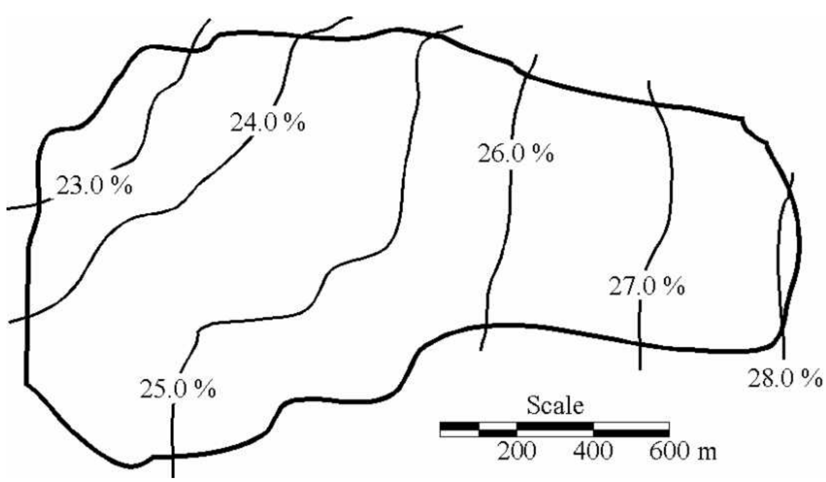

FIGURE 8
CONTOUR MAP OF THE OIL-BEARING FORMATION SHOWING THE POROSITY OF THE FORMATION.

5. Oil usually does not occupy the total pore volume of a formation. It usually shares the pores with natural gas and water which are normally found wherever oil is found. The fraction of the pore volume occupied by the oil is usually known as the oil saturation at it is normal to express it as a percentage. If the oil occupies two-fifths of the pore space then the oil saturation is 40 %. The variation of oil saturation throughout the formation is shown below. Estimate the average oil saturation for this formation. Using the above information above estimate the actual volume of oil contained within the reservoir.

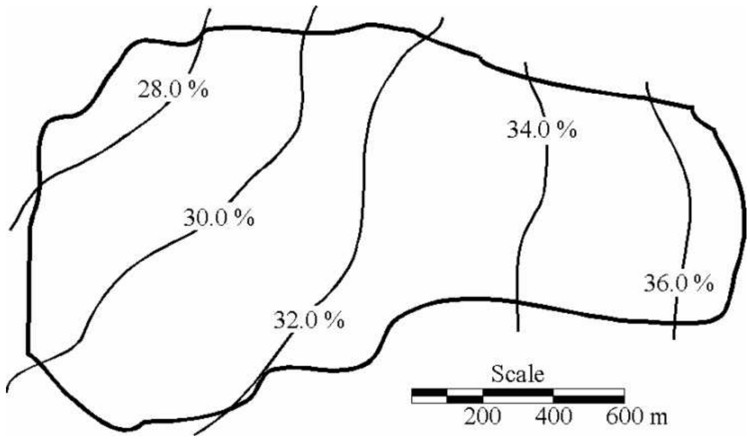

FIGURE 9
CONTOUR MAP OF THE OIL-BEARING FORMATION SHOWING THE OIL SATURATION OF THE FORMATION.

6. In many cases the oil and water found in petroleum reservoirs will flow to the surface unassisted. This is because the pressure within the oil reservoir is sufficiently high. As the oil and water are produced however the pressure within the reservoir will usually begin to decrease and consequently the flow rate of oil will begin to decline. At the same time the number of barrels of water produced by a well per barrel of oil will begin to increase. It is not unusual for an oil well to produce two barrels of water per barrel of oil. When this ratio increases to, for example, eight to one, then action must be taken by the petroleum engineers.

   Table 3 presents data for the production rate of oil as a function of time for a particular well. Also presented is data for the water to oil production ratio (known as the WOR). This data is shown in Figure 10.

   Use this data to estimate when the production rate of water increases for the first time above 500, 750 and 1000 barrels per day. Plot the production rate of water as a function of time. Fit a decay function to the oil production data.

CONCLUDING REMARKS

It is not possible to quantify the increase in interest in engineering amongst secondary school students as a result of the use of these and other similar activities in class room. Anecdotal evidence suggests that however that they have been useful in demonstrating that engineering is broader than just civil engineering.

The author hopes that readers will be encouraged to develop this and other activities further for use in class rooms in their home locations.

ACKNOWLEDGEMENT

The author acknowledges the work of Annette Schwarz who conducted the heaps experimental program.

TABLE 3
OIL WELL PRODUCTION DATA

Time (weeks)	Oil Production Rate (barrels per day)	Water to Oil Production Ratio
0.00	243.00	1.72
1.00	239.00	1.74
2.00	240.00	1.78
3.00	220.00	1.77
4.00	225.00	1.85
5.00	213.00	2.02
6.00	209.00	2.20
7.00	178.00	2.50
8.00	186.00	2.94
9.00	173.00	3.50
10.00	172.00	4.10
11.00	178.00	4.22
12.00	174.00	4.81
13.00	171.00	5.43
14.00	167.00	6.50
15.00	169.00	7.75
16.00	166.00	8.20
17.00	168.00	8.70
18.00	166.00	9.30

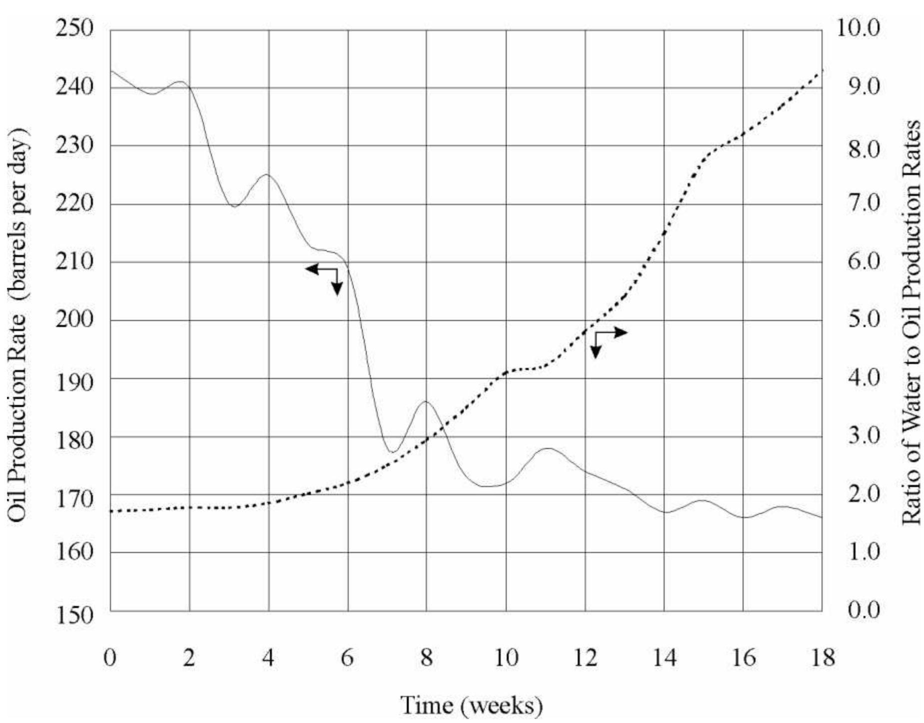

FIGURE 10
PLOT OF OIL PRODUCTION RATE AND WOR AS A FUNCTION OF TIME. SOLID CURVE IS OIL PRODUCTION RATE, DASHED CURVE IS WOR.

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