EDUCATING MATHEMATICALLY-DEFICIENT ENGINEERING STUDENTS

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Abstract *¾ Evidence is presented of the mathematics problem facing engineering departments and a possible solution is discussed based on a new teaching approach that puts engineering first and mathematics second without impacting negatively on standards. Described as minimal-mathematics, this approach requires a fresh look at the derivation and presentation of old engineering concepts and principles in order to subordinate mathematical rigour to an unclouded insight into the engineering problem. This can be coupled with computer-integrated lecturing to create a multi-sensory engagement of the student and transform a potentially boring chalk-and-talk session into the fun-filled experience that most new intakes have come to associate with computers and television. The paper includes detailed illustrative examples drawn from the field of telecommunications and reports of positive feedbacks from students on which the approach was tested.*

Index Terms ¾ Engineering education, matched filter, mathematical skills, sampling

INTRODUCTION

Engineering academics in the UK are fairly unanimous in acknowledging that there has been a noticeable decline in the quality of new intakes into various engineering programs in universities. A growing number of new entrants are deficient in important mathematical skills, have poor problem-solving discipline and are ill at ease with the traditional chalk-and-talk teaching approach. Some engineering departments experience an unacceptably high drop-out rate amongst first-year students. In one university observed during the 1999/2000 academic year, the School of Electronics had by far the highest drop-out rate of 29% of first-years. Many engineering departments face a real problem of student recruitment and retention. How do you entice young people onto a program of study that is negatively perceived as highly mathematical? Having somehow recruited, how do you retain the vulnerable ones long enough to remedy their deficiencies and endear them to the study of engineering?

Confronting weak students with a lot of mathematics before they have had the opportunity to remedy their deficiencies and improve their attitude is likely to lead to withdrawal or failure. On the other hand, engineering modules on offer cannot be purged of mathematical content without seriously diluting the depth of treatment and therefore significantly undermining the competence of graduate engineers.

First, the paper briefly provides some evidence of the mathematics problem. Observation of an overwhelming preference for minimal-mathematics amongst successive generations of students taking a first-year telecommunications module is reported. This gives an indication of mathematical attitude rather than ability and the results spanning a three-year period consistently reveal a worryingly negative attitude to mathematics. Evidence for the alleged declining mathematical ability amongst new university intakes over the last decade comes from two independent and objective diagnostic tests conducted by others at two UK universities. The paper then suggests a way of successfully training engineering students who are initially deficient in mathematics by eliminating (without necessarily lowering standards) the mathematical high-hurdle inherent in traditional teaching approaches. The suggested minimal-mathematics approach treats mathematics as a useful tool rather than an end, and seeks first of all to win new students over to the sheer enjoyment of an engineering study, allowing a department more time to remedy their mathematical deficiencies with less risk of failure or withdrawal. The approach is illustrated using two important examples drawn from the engineering field of telecommunications.

MINIMAL-MATHEMATICS PREFERENCE

A measure of mathematics attitude has been taken since 1998 amongst successive cohorts taking a first-year module dealing with basic telecommunications in the school of Electronics at the University of Glamorgan in the UK. In one question the students were asked to choose one of three options to indicate what they would prefer in the module: (A) Mathematics only where unavoidable; (B) A lot of mathematics; (C) Absolutely no mathematics. Another question sought to find out what the students would do if there was a lot of mathematics in the module. Three options were provided: (A) Take on the challenge happily; (B) Endure; (C) Withdraw.

Figures 1 and 2 show the response in terms of the percentage of students choosing each option during the three academic years. Clearly, the vast majority of these students are averse to much mathematics. In the three cohorts questioned, an average of only 5% of the students (with a standard deviation of 2%) wanted a lot of mathematics in the module. Worryingly, an average of 39% of the students wanted absolutely no mathematics. An average of only 27% of the students would happily accept the challenge of a lot of mathematics in the module. Note however that this particular

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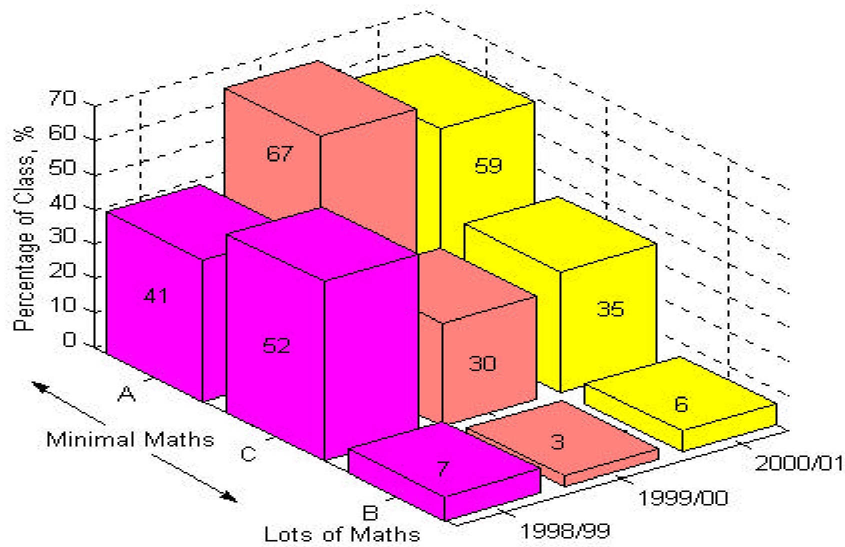

FIGURE 1
PERCENTAGE OF STUDENTS SELECTING EACH OF THE OPTIONS: (A) MATHS ONLY WHERE UNAVOIDABLE; (B) A LOT OF MATHS; (C) ABSOLUTELY NO MATHS.

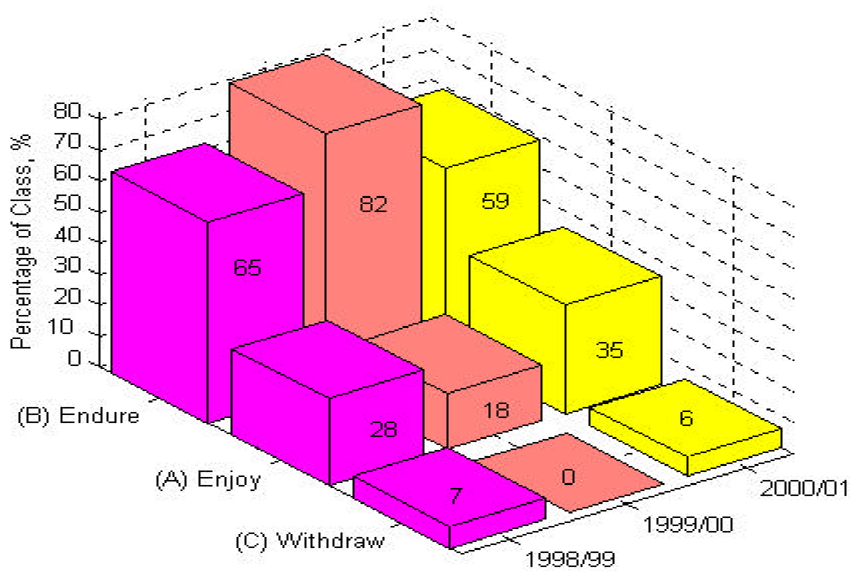

FIGURE 2
PERCENTAGE OF STUDENTS WHO, IF THERE WAS A LOT OF MATHS, WOULD (A) TAKE ON THE CHALLENGE HAPPILY; (B) ENDURE; (C) WITHDRAW FROM COURSE.

group of students are not necessarily mathematically skilled— as evidenced by responses to another (unreported) question in the questionnaire. It can be seen that a mathematical approach would be off-putting to about three quarters of the students. Ignoring their preference can contribute to avoidable attrition. Interestingly, when the first question was asked to 13 practising engineers— products of various UK universities, some of them with less than 2 years experience— attending a 5-day short course on digital telecommunication networks held in October 2000, eleven of them (85%) categorically chose option (A)— Maths only where unavoidable, with the remaining going for a lot of Maths (option B).

One must therefore question the wisdom of continuing to unleash on new engineering students a mathematical approach from a golden age of mathematics, when the vast majority of these students clearly belong to a different age. Some academics fear that standards would fall if they abandoned the 'good old' approach. It is however the assertion of this paper that it is possible to accommodate the observed minimal-mathematics preference of the current breed of engineering students without compromising standards. Furthermore, modernising our teaching approach would make an engineering study look feasible to an unquantified fraction of pre-university students who are currently perhaps too scared of mathematics to contemplate the prospect.

DECLINING MATHEMATICS ABILITY

A recent report [1] published by the UK Engineering Council presents a number of objective pieces of evidence for declining mathematical skills amongst new university intakes. Two of these are summarised below.

- The performance of new undergraduate physicists at the University of York in the same diagnostic mathematics test administered every year since 1979 is similar until 1990 when there is a sharp drop followed by a steady decline over the past decade. In particular, while the average score of the 1986 intake was 76%, that of the 1997 cohort was only 50%, and none of the intakes since 1995 have registered an average score above 56%.
- Standard diagnostic tests have been given to new students entering Coventry University's mathematics-based courses since 1991. Grouping the students according to their A-level Maths grade, the results show that the performance of students with the same A-level grade has declined steadily over the years, signifying a dilution in grade by about one grade every two years.

The general consensus is that students with a good A-level mathematics grade can no longer be assumed to possess the mathematical skills required by the traditional approach in the training of engineers. Engineering departments have a greatly reduced pool of students with adequate mathematical preparedness from which to recruit into their degree places. New intakes in many departments now come from a wide range of vocational, A-level, mature, access and foundation backgrounds. This gives rise to a very inhomogeneous cohort and consequently a mixed-ability teaching due to staffing and accommodation constraints.

Diagnostic testing of new undergraduates is now widely used to identify the mathematical weakness of individual students and that of the whole cohort. Remedial measures can then be individually prescribed to bring each student up to speed. However, until there is a critical review of how a mixed-ability group with strong preference for minimal-mathematics is taught, the wastage rate amongst engineering students is likely to remain high as many of the mathematically vulnerable students may join the growing list of casualties before remedial measures have had time to yield the intended benefits.

THE MINIMAL-MATHEMATICS METHOD

Urgent action is needed to increase retention of mathematically deficient intakes in engineering departments, but adopting a non-mathematical curriculum is out of the question since this would seriously undermine the competence of graduate engineers. A new approach can however be considered that puts engineering first and mathematics second, and gives beginning students the opportunity to become endeared to engineering study while gradually remedying their deficiencies in mathematics. Wastage statistics suggest that students are most at risk in their first year. Thus, there is a pressing need for a review of the traditionally mathematical approach in the delivery of first-year engineering courses.

The best interests of the vast majority of students and a good engineering education can be jointly served through a minimal-mathematics approach where mathematics is employed only to the extent and at a level that is necessary. Emphasis is placed on the underlying engineering considerations, and lucid graphs and diagrams are employed to facilitate understanding. Mathematics is given its rightful place, which is second to engineering and a physical insight into the problem at hand. A graphical approach is freely used where necessary to simplify a mathematical computation or illustrate a theorem. Developing such a teaching approach requires a thorough reassessment of current methods, changes in approach being made wherever necessary to meet students at their level while ensuring that they have an unclouded insight into the underlying engineering concepts and a good grounding in the theory.

It is insufficient to string together an elegant mathematical derivation of a concept for students who do not yet possess the skill or motivation to follow. Such an approach is rather insensitive and leads to students employing 'black-box' formulas whose limitations they cannot fully appreciate. They will lack the insight, confidence and competence that could very easily have been gained from a minimal-mathematics approach. This approach will now be illustrated with two important examples drawn from the field of telecommunications.

Matched Filter: The Minimal-Mathematics Approach

The specification of a receive filter (called a *matched filter*) that gives optimum detection of a signal in the presence of additive white Gaussian noise is usually derived by invoking Schwarz's inequality. In Figure 3, the transfer function $H(f)$ of a filter is to be specified that maximises the instantaneous output signal power $P_o(T_s)$ compared to the average output noise power P_n, where T_s is the sampling instant or observation interval.

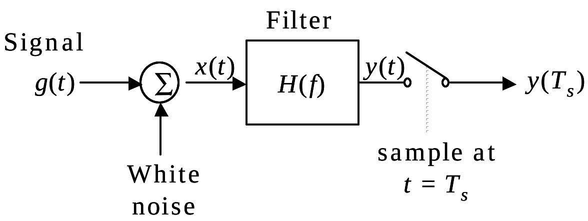

FIGURE 3
SIGNAL DETECTION IN NOISE

The usual approach for deriving $H(f)$ is very mathematical and is presented in detail elsewhere [2]. Let us consider a minimal-mathematics approach for determining $H(f)$ and hence the matched filter's impulse response $h(t)$ by making three increasingly prescriptive observations.

- The bandwidth of the filter must be just enough to pass the incoming signal. If it is too wide, noise power is unnecessarily admitted, and if it is too narrow then some signal energy is cut out. Thus, the input signal spectrum $G(f)$ and the filter's transfer function $H(f)$ must span exactly the same frequency band. How should they be shaped?
- The gain response $|H(f)|$ of the filter should not necessarily be flat within its passband. Rather, it should be such that the filter attenuates the white noise significantly at those frequencies where $G(f)$ is small— since these frequencies contribute little to the signal energy. And the filter should boost those frequencies at which $G(f)$ is large in order to maximise the output signal energy. Therefore the filter should be tailored to the incoming signal, with a gain response that is small where $G(f)$ is small and large where $G(f)$ is large. In other words, the gain response of the filter should be identical in shape to the amplitude spectrum of the signal. That is,
$$|H(f)| = K|G(f)| \qquad (1)$$
where K is a constant.
- A complete specification of the filter requires its phase response. This is accomplished by noting that the maximum instantaneous output signal power occurs at the sampling instant $t = T_s$ if every frequency component (i.e. cosine function) in the output signal $g_o(t)$ is delayed by the same amount T_s and has zero initial phase so that
$$g_o(t) = A_o \cos[2\pi f_o(t - T_s)] + A_1 \cos[2\pi f_1(t - T_s)] + A_2 \cos[2\pi f_2(t - T_s)] + \cdots \qquad (2)$$
resulting in a maximum instantaneous value at $t = T_s$ given by $g_o(T_s) = A_o + A_1 + A_2 + \ldots$
where $A_o, A_1, A_2, \ldots$ are the amplitudes of the sinusoidal components of $g_o(t)$ of respective frequencies $f_o, f_1, f_2, \ldots$. Note that these frequencies are infinitesimally spaced, giving rise to a continuous spectrum $G_o(f)$. Rewriting (2) in the form
$$g_o(t) = A_o \cos(2\pi f_o t - 2\pi f_o T_s) + A_1 \cos(2\pi f_1 t - 2\pi f_1 T_s) + \cdots$$
shows that the phase spectrum of the output signal $g_o(t)$ is $\phi_o(f) = -2\pi f T_s$. And since $\phi_o(f) = \phi_H(f) + \phi_g(f)$, where $\phi_H(f)$ is the phase response of the filter and $\phi_g(f)$ is the phase spectrum of the input signal, it follows that
$$\phi_H(f) = -\phi_g(f) - 2\pi f T_s \qquad (3)$$
Combining (1) and (3) gives the required filter transfer function,
$$\begin{aligned} H(f) &= K|G(f)|\exp[j\phi_H(f)] \\ &= K|G(f)|\exp[-j\phi_g(f)]\exp(-j2\pi f T_s) \qquad (4) \\ &= KG^*(f)\exp(-j2\pi f T_s) \end{aligned}$$
The impulse response $h(t)$ of the filter is the inverse Fourier transform of its transfer function $H(f)$, and follows from equation (4) when it is noted that complex conjugation of $G(f)$ corresponds to a time reversal of the real signal $g(t)$; and that multiplying $G^*(f)$ by the exponential term $\exp(-j2\pi f T_s)$ corresponds to delaying $g(-t)$ by T_s. Thus,
$$h(t) = Kg(T_s - t) \qquad (5)$$

Thus the impulse response of a matched filter that gives optimum detection of a pulse $g(t)$ in the presence of white noise is simply a time-reversed and delayed version of the pulse. Note that compared to the traditional approach, the above derivation places emphasis on a physical insight into the problem and is significantly toned down in mathematical rigour— an appealing feature to most students.

Analogue Signal Sampling: The Minimal-Mathematics Approach

The process of sampling is pivotal in the digital transmission of audio and video signals. Traditionally, sampling is covered using a mathematical approach that employs the Dirac delta function, Fourier transform, integrals and a sinc interpolation function to derive conditions for a distortion-free sampling and discuss the process of reconstructing the original signal from the samples. Reference [3] however treats this topic with great clarity using a minimal-mathematics approach to impart to the student a thorough understanding of all the important concepts and a proficiency in anti-alias filter design, aperture effect correction, sampling of lowpass and bandpass signals, etc. Note for example how Figure 4 illustrates that the samples of a sinusoid of frequency f_m taken at a rate f_s samples/second could originate not only from the original sinusoid, but also from an infinite array of sinusoids of frequencies $nf_s \pm f_m$, where n = 1, 2, 3, … In other words, the spectrum of the sampled signal contains the frequencies

$nf_s \pm f_m$, n = 0, 1, 2, 3, … That is, (instantaneous) sampling takes the baseband frequencies $\pm f_m$ of the sinusoid and replicates them without change at regular intervals f_s along the frequency axis. For an analogue signal comprising more than one sinusoid ($\pm f_1$, $\pm f_2$, $\pm f_3$, …), each frequency component is separately replicated. This is illustrated in Figure 5 where it is clear that the samples of signal ($\pm f_1$, $\pm f_3$) contain an infinite array of components at $nf_s + (\pm f_1, \pm f_3)$, n = 0, 1, 2, 3, …. In general therefore, the effect of sampling an arbitrary information signal of bandwidth B is illustrated in Figure 6. It follows that so long as $f_s \geq 2B$, the undistorted baseband spectrum (n = 0), and hence the original signal, can be recovered from the samples by passing them through a low pass filter, which passes the spectrum at n = 0 but blocks all the spectra replicated at $n \geq 1$. Sampling at $f_s = 2B$, known as the Nyquist rate, requires an ideal brickwall filter (with no transition band) for distortion-free reconstruction. If however $f_s < 2B$, there is some overlap between duplicated spectra. The resultant spectrum is thereby distorted and a low pass filter is no longer able to reconstruct the original signal. This distortion is known as aliasing. The underlying cause of aliasing dis tortion is illustrated in Figure 7 where we see that if we sample a sinusoid f_m at a rate f_s which is less than two samples per period, then the samples contain (i.e. fit) a lower frequency sinusoid of frequency $f_a = |f_s - f_m|$. A low pass filter will unavoidably pass (i.e. reconstruct) this alias sinusoid f_a along with f_m, and it is the presence of this new frequency component that causes a distortion of the reconstructed signal.

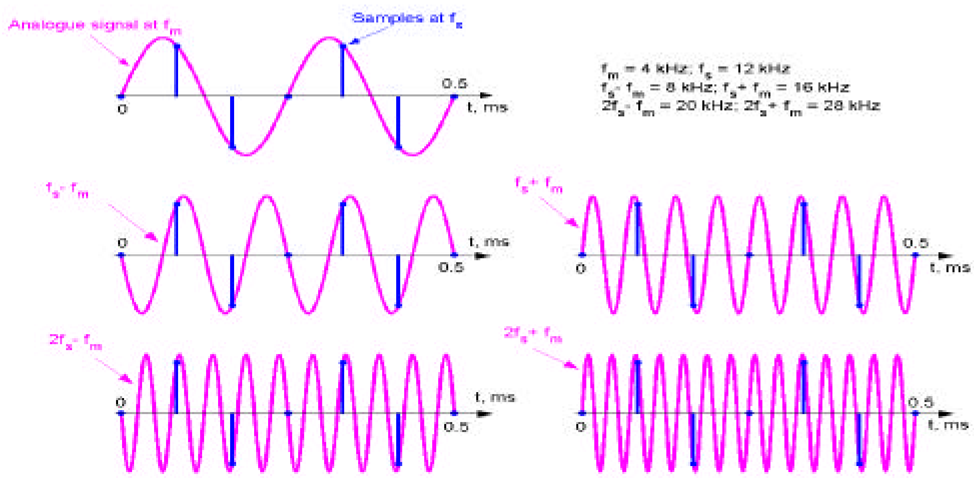

FIGURE 4
SAMPLES OF A SINUSOID F_M TAKEN AT RATE F_S ALSO CONTAIN (I.E. FIT) THE SINUSOIDS $NF_S \pm F_M$, N = 1, 2, 3, …

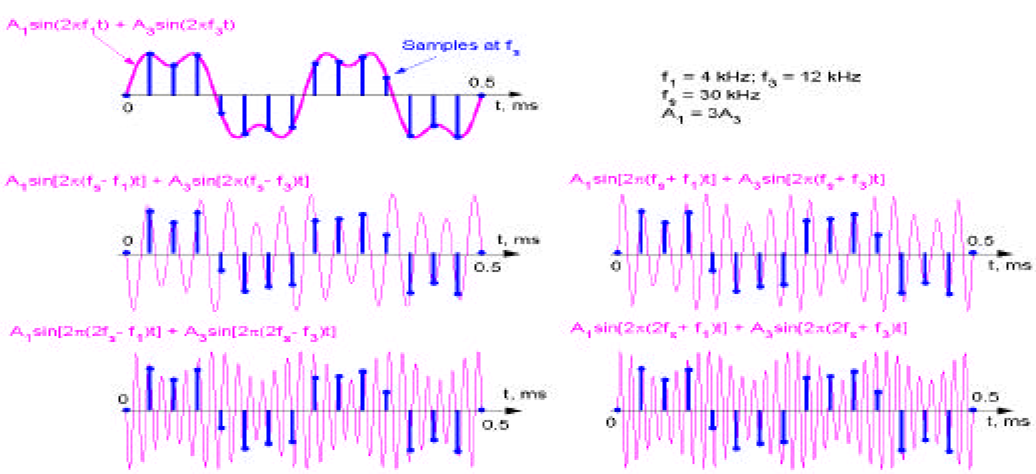

FIGURE 5
ARRAY OF FREQUENCY CO MPONENTS CONTAINED IN THE SAMPLES OF AN ANALOGUE SIGNAL CONSISTING OF 2 SINUSOIDS

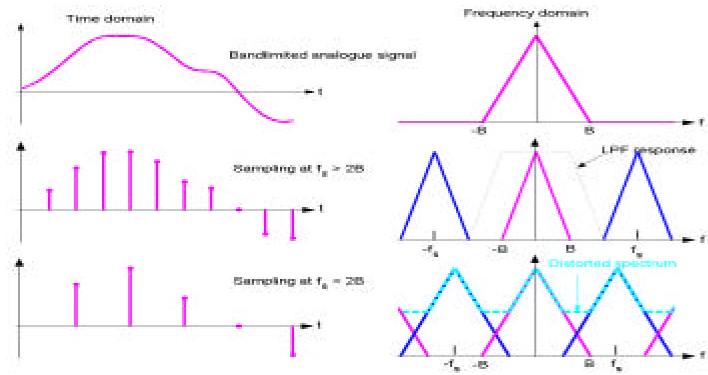

FIGURE 6
SAMPLING A BANDLIMITED ANALOGUE SIGNAL

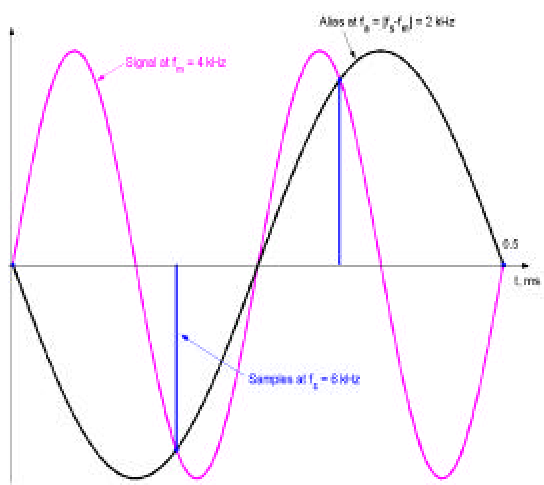

FIGURE 7
ALIASING DUE TO UNDERSAMPLING AT $F_S < 2F_M$

In the foregoing, we have introduced important concepts— sampling theorem, Nyquist rate, aliasing, etc— without the use of mathematics. In fact the discussion can be successfully extended [3] beyond instantaneous sampling without any need for abstruse mathematics. This minimal-mathematics approach imparts to the student an unclouded insight into the problem and is ideally suited not only to the mathematically-deficient student, but also to the more skilled student when first encountering the subject.

CONCLUSION

The widely acknowledged declining mathematical skills of university entrants and their penchant for minimal-mathematics necessitate a critical review of the teaching approach employed by engineering academics to cater to the ability, experience and preference of a new breed of intakes. Since wastage statistics show students to be most at risk in their first-year, it is particularly important that engineering departments explore ways of modernising the way first-year students are introduced to the subject in order to eliminate what a growing number of new and potential recruits perceive as an unfriendly mathematical gatekeeper.

In this paper a minimal-mathematics approach was proposed and detailed examples of its use were presented. Innovative thinking and further research can lead to the extension of this approach to more and more engineering topics that are traditionally wrapped in mathematics. The approach puts engineering first and mathematics second, giving students an excellent insight into each engineering problem and equipping them to use mathematics wherever necessary as a problem-solving tool rather than an end. This innovative approach can be combined with computer-integrated lecturing, whereby students first interactively 'discover' key parameters and concepts through guided computer simulations and a lecturer then hangs a more detailed knowledge on the pegs that have been thus erected. The result would be the transformation of potentially boring or daunting lectures into a multi-sensory and fun engagement of the students. Engineering education would benefit in terms of a significant improvement in recruitment and retention figures as well as the confidence of the finished products.

The method espoused in this paper has been tested in short courses on telecommunications for practising engineers and attracted very positive feedbacks such as: 'Made the subjects enjoyable and easy to follow'. 'Excellent analogies and examples were used to describe complex theorems'. 'Explained very abstract subjects in a way that improved understanding of mathematical descriptions'. 'Fantastic'.

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