ENGINEERS NEED MATHEMATICS BUT CAN WE MAKE IT INTERESTING?

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***Abstract* ¾** *It is well accepted that Engineers need mathematics. However, engineering students find mathematics difficult, boring, dull and much of the time they perceive it to be irrelevant. In the Department of Electronics and Electrical Engineering at University of Glasgow we have been using a project based approach in the delivery of mathematics to first year students. By doing so we believe that they will develop their mathematical abilities and additionally develop their social skills. The students on this course work on the mathematical analysis of real engineering problems and the applicability of engineering and mathematics to areas outside the remit of a conventional first syllabus. They work together in groups, so developing their social skills and additionally they are assessed by individual oral presentations which help develop their communication and linguistic skills. In the paper we will describe and analyse the success of the project based approach, looking in particular at the projects that have been developed specifically for this course.*

Index Terms- Project Based Approach, Mathematics teaching, skill development.

INTRODUCTION

Engineering students find mathematics difficult, boring, dull and much of the time they perceive it to be irrelevant. However it is our contention and that of others [1] that Engineers need mathematics. Mathematics is the foundations upon which all engineering analysis is built. This need for a sound base of mathematical skills coupled with the disaffection of students towards a mathematical approach to teaching has, at our institution, adversely affected the student retention rate. Two other influences bear upon any deliberations on the teaching of mathematics to engineering undergraduates. Firstly, many teachers in higher education (particularly in the UK) bewail the level of mathematics taught in schools. However, on arrival at university students do not wish to work through pages of examples to develop the manipulative skills they need. Secondly, over the last twenty years the context in which engineers use mathematics has changed. Calculations that used to be done by hand are today solved by an engineer selecting an appropriate software package. The skills and method of analysis used by engineers in such situations are radically different to those needed historically.

In common with engineering Departments throughout the world, members of the Department of Electronics and Electrical Engineering at the University of Glasgow, working with the Educational Developers in the Teaching and Learning Service have devoted a great deal of time in recent years to addressing these issues, [2] – [4]. In this paper we will briefly describe the format of the first year course and then concentrate on the contribution that projects have made both to the mathematical understanding of students and also to their development as engineers.

LEARNING OBJECTIVES

At the outset we wish to be clear about the outcomes we expect from students by the end of the first year in terms of mathematical ability. The general learning objectives (which are, in our experience, common to most engineering mathematics courses in the UK)- can be stated to be that the students will learn to:

- perform simple manipulative skills
- read and interpret mathematical texts
- select and use software to perform calculations
- apply mathematics in the solution of engineering problems.
- develop strategies for solving extended problems.
- explain the meaning of mathematical expressions and the mathematical solutions to engineering problems in a clear and logical way both in writing and orally
- work with colleagues to solve mathematical problems, share information and ideas.

The challenge for us was how to design and deliver a module which inculcated the knowledge, skills and understanding needed to meet these objectives in our students.

PHILOSOPHY OF THE COURSE

In designing the course we examined a number of options in terms of models to use, but in the end we settled for what can be described as a 'project-based' approach. This is in some ways a compromise resulting from the difficulties of innovating in a research led university. At one extreme of a spectrum utilizing a problem based learning approach may well have been a preferred choice for some of the department, others saw no need for change, hence a

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project based approach was a half way house that the majority of the staff felt comfortable with.

To us the advantages of a project based approach are:

- Motivation is improved, since learning is taking place in a situation more closely related to 'real' engineering
- There is inevitably a logical selection process with regard to projects to reinforce those parts of the curriculum which are most important.
- The ability to study in an independent manner is nurtured from the outset.
- The environment demands that the student works with colleagues, unlike more conventional courses where, collaboration, while not frowned upon, is seldom deliberately encouraged.
- Skills can be developed by students at the same time as learning the academic subject, i.e. leadership skills, interpersonal skills, presentation skills are all important to an engineer and can be developed using this approach.

However, the project based approach has disadvantages, namely:

- The possibility that studies may be highly specialized and not encompass all of what is thought to be important (after all can only use a limited number of projects)
- Having handed over autonomy to a group on an open ended project, the teacher has little control over their subsequent actions and sometimes the decisions made are unsound.
- Practical and applied topics may be favoured at the expense of important theoretical topics.
- Assessment of individuals is problematic if the work is done a group.

However after careful consideration of the advantages and disadvantages of this approach it was decided that it was worth while adopting the approach, and evaluating its effectiveness as a teaching philosophy.

DESCRIPTION OF THE COURSE

We will describe the course in some detail because the projects are not stand alone but embedded in a frame work which both develops other skills and supports the knowledge and skills required for a successful project. We do not believe that the whole course can be based on project work, because as musicians practise scales, mathematicians also have to develop pattern recognition and the ability to perform simple tasks - to build up a ubiquitous base of techniques which are applicable to a wide range of problems.

Mathematics accounts for a quarter of the first year curriculum. Five hours a week are taken up with staff contact through lectures, tutorials, computer aided learning packages and tests. Students are expected to work for at least 8 hours per week on this course outside formal classes. In order to encourage them to engage with mathematics to this level, particularly when there are many distractions both in their other course work and from their social life outside University, involves trying to both enthuse them and also to put in the appropriate levels of assessment to drive appropriate learning.

The course has been designed to recognise these problems associated with student learning and address them on several fronts. It is divided into four blocks of six weeks each. In each block there is a project, an assigned tutorial and a class test.

In the first four weeks of each blocks there are 11 lectures and four computer aided learning sessions. The four weeks end with a 1 hour driving test which is aimed at developing the basic skills and manipulation the student should achieve. If the students do not achieve an A grade (70%) in this test they have to retake it four weeks later. The tests counts 20% towards the final mark for the course. There are skills exercises for each of the first four weeks and the students are recommended to do one exercise every day which should take them about 1/2 an hour. This is based on a philosophy that doing a little often rather than a lot infrequently is more likely to result in the material being long term understood. The driving tests are based on these exercises.

In addition to the skills exercises there is an assessed tutorial which contains longer problems, based on engineering applications. Students are expected to give in a clearly presented piece of work. They are encouraged to discuss the solutions with other members of their tutorial group and their friends. The material may not have been covered in the lectures and they are expected to read the text books to find a solution. Discussions with their colleagues and consulting relevant texts are essential engineering skills. However for this exercise they must give in their own solution and not copy any material unless it is clearly and appropriately referenced

Each week the students have a timetabled slot in the computing laboratory. In the first four weeks they access computer-aided learning (CAL) packages which re-inforce the course material for that week. The main package we use is MathWise, a teaching package developed by a consortium of Universities in the UK. This is supplemented by CALMAT, a more elementary teaching package.

In the last two weeks of the course the students have no lectures but concentrate on their projects.

They work on specified problems which need mathematics, but also contain elements of engineering. The results are presented at the tutorials but the mark for this part of the course is assessed through oral presentations in February and May. In each project the students use MathCad, a general-purpose program for symbolic manipulation and the visual display of mathematics.

STAFF CONTACT FOR STUDENT SUPPORT

The relationship between the students and the staff is an essential part of the learning environment. While lectures and CAL material can be delivered efficiently to large numbers of students - we typically teach classes of 100 students, we believe that reflective and critical skills, at the core of any learning, but particular necessary for mathematics, can only be engendered in small groups. Therefore every student is assigned to a tutorial group of eight students, facilitated by an academic, which meets weekly throughout the year. The tutors encourage the students to work together to solve problems, share knowledge and communicate their findings to the group. In addition the same group has a mentor, a post-graduate student or research assistant. The role of the mentor is to ease the transition between school and University for the first year students. However, since mathematics is part of the core of teaching they work with the tutors, giving formative orals and discussing the projects with the students. Most students have had little previous practice with group work. There is no time for discussion about how to go about it in the first few weeks of term. Since these are obviously skills they will need to master, it is rather unfair to expect them to work harmoniously with no preparation We address this by running a workshop with the mentors. The hope is that by warning the mentors and discussing the kinds of problems their groups were likely to face, they would be better prepared to help the students. This also follows the industrial practice of cascading down information so both the mentors and the students benefit.

THE PROJECTS

Overall Aims

All the projects involve some aspect of mathematical modelling. We try to make the students stop, think about and describe what is happening in a physical situation. In each of the projects, there is a small experiment or demonstration which should help them visualise the problem. This happens before they use mathematics as a (symbolic) language to describe the patterns or system. Only after this do they use software (here MathCAD) to play with and analyse the equations/language. This structure moves the students from considering an actual physical system, to developing a mathematical description, to a deeper exploration using software. Each step reinforces the previous ones, encouraging reflection and promoting a cycle of learning.

It is vital that, as engineers, students recognise that mathematics is a powerful tool. However, there is a danger in leading students to believe that an ability to do algebraic manipulations, solve equations, and plot graphs is the entirety of 'engineering'. We want to encourage them to play - we ask them to 'play' with a system - to come up with an 'experiment' or play with a demonstration. Students become excited if involved - they like to discuss ideas, play with simple experiments and the play with the software. Success is if there is a buzz of discussion and quiet laughter in the laboratory. Laughter in the classroom is a powerful tool for learning. This 'playing' is smaller scale and much less formal than traditional labs which teach different skills. We believe that it is also vital that engineers learn how to investigate something simply –the arithmetic growth of an Archimedes spiral can be described as the pattern traced out when a tethered horse unwinds itself from the tree it had wrapped the rope around. To draw one needs nothing more than a coke can and a shoe-lace!

Finding projects at a level to engender this kind of learning is not easy. They cannot be too trivial or the students lose interest. They must not be too difficult or the students loose confidence. Everyone in the group must able to participate and contribute to the discussion. This is not an easy remit and we are still improving our design.

In making the project enjoyable we must not lose sight of the fact that learning is not easy. We are trying to create the situation in which students want to work; where they play with the software packages and are excited about their ability to display and solve equations. However, it is vital that in order to use these packages as a tool, the students become familiar with the syntax and power of software. As with the skills exercises to develop basic manipulative skills, it is easy to achieve this in isolation, teaching only repetitive skills. But following our philosophy of embedding mathematics in an engineering environment, we want to reinforce those skills whilst emphasising understanding and the ability to communicate that understanding. The projects have been constructed so that the time they spend using MathCAD will deepen their understanding of the system and mathematics in question by allowing them to find ways to use the computer to do repetitive calculations, plot graphs etc. The group work in the projects is therefore balanced with a set of individual exercise to help students up the learning curve of Mathcad We have designed a set of basic exercises they try before the project.

Also to increase the students interest and link the projects to the wider world, each project description contains excerpts of background information and references to relevant web-sites. These are varied and aimed at interesting different sorts of students - for instance when learning about complex numbers some students may be interested in discussing that the terms 'complex' and 'imaginary' are somewhat unhelpful and even controversial! Calling them 'normal' numbers as Dresden suggested has very different connotations. Others get excited when they discover that the bones in their hand and body are roughly in a Fibonacci sequence and this can be explained by a mathematical model.. The aim throughout is to make mathematics relevant to engineering but fun in its own right!

Project One

The first block revises algebraic manipulations, functions, partial fractions, sequences and series. The real challenge here is to encourage the students to see patterns in mathematics and to begin to think about how mathematics can be used to describe the world around them. We use the Fibonacci series as an example of a recurrence relation. It is a very simple yet powerful pattern, one that is prevalent in nature and human constructs.

The project consists of a number of parts, which are for this first one, designed to help move the student away from following a 'recipe book' to beginning to develop the skills of 'I wonder would happen if I did ….'

- Archimedes spiral: they are asked first to construct one to think about the mathematical description, then to draw one with MathCAD so that they can test the effect of varying the parameters.
- Logarithmic spirals: in what way is the pictured spiral different from the one they have been working with? Then think about the mathematical description and again draw it with MathCAD
- A description of the Golden Ratio, f, and its relation to the Fibonacci sequence. Students to show that the ratio of successive terms in the Fibonacci series converges to f.
- A summary of what has been covered. The last section asks them to choose one phenomenon influenced by f and/or the Fibonacci series and to explore it.

Project Two

Block 2 concerns complex numbers, vectors and matrices. Here we felt one of the most important aspects was a conceptual understanding of complex numbers. It is too easy for (engineering) students to get bogged down in the abstract definition $j = \sqrt{-1}$ and not appreciate how useful complex numbers can be in engineering applications. Likewise we wanted to emphasise a practical use of matrices.

For Electrical Engineering students the most obvious application for complex numbers is in describing a.c. circuits. We therefore collaborated with the lecturers on the first year electronics course to find an example that was relevant to both courses.

The elements of this project are

- Background to complex numbers, particularly discussion of their name. (Dresden suggested calling them 'normal numbers')
- Discussion of complex numbers as vectors and an exercise to reinforce the concept.
- Discussion of the benefits of using complex numbers in the description of a.c. circuits.
- Mathematical analysis of a radio tuning circuit. This is a circuit they build and test as part of their Electronics lab at around the same time. Here they have the chance to look at it mathematically.

Calibrating a platinum resistance thermometer. In their electronics course they have been doing nodal analysis of circuits. Here they do a more complicated one and have to choose values of components based on their analysis. This brings in the knowledge from the Electronics course, matrices, MathCAD and a feel for engineering – when is something linear? How large a resistance can the circuit sensibly deal with?

The student really enjoy constructing a circuit, however simple. Tying the mathematics into this activity generates questions and discussions. Electronics is what most students want to do- getting them to see that mathematics can be an interesting and informative part of this activity engenders an acceptance of mathematical analysis and a willingness, on the part of many students, to put in the work to get over the pain barrier and become confident of their ability to use mathematical techniques.

Project Three

In block 3, the students are introduced to calculus. By the end of the block they should be able to differentiate and integrate a range of reasonably straightforward functions. We decided to use this project to illustrate and develop some of the research skills needed to solve engineering problems. We also felt that the students should explore a classic problem - there are a number of classic problems that are relevant to many situations and that should be a part of every students toolbox. One of these are the equations for growth of limited resource. As an example of this, the students are asked to look into the factors influencing oil production rates and produce three different mathematical models for the trends based on different (specified) assumptions. They are given excerpts from a variety of sources, asked to find the necessary information for themselves, and comment on the outcomes of the predictions. This analysis can used to predict population growth, use of mobile telephones, effects of foot and mouth disease etc.

Project Four

Block 4 looked at applications of calculus: critical points and optimisation and linear first and second order differential equations. We wanted to illustrate how far-reaching the general differential equations are and to show that each of the terms describes a specific behaviour. Students are asked to analyse the second order linear differential equation describing the motion of a simple pendulum term-by-term. At each stage they have to consider how to produce the equivalent behaviour in an electronic circuit. The last section asked the groups to think of other systems which could be described with these equations. Again excerpts from books and papers give them a starting point and they are encouraged to find other references and web-sites.

Student Response to Project 1

Overall the students seemed to enjoy and benefit from the project. One student in the tutorial group said that he hadn't previously realised that *"maths isn't just numbers, it's everywhere …"*

Another focus group had all enjoyed the project but some claimed to have found it easy. It transpired, however, that some of them had inputted the Fibonacci sequence directly rather than programming MathCAD to generate it for them. This finding led to a discussion of how much the tutors should check the students' work. For example, will reminding the students regularly that they will need to be able to use MathCAD in their 2nd year sufficiently encourage them all to learn it? It had been assumed that the tutor's role was to check the students' understanding and to encourage them to practise presenting their work orally. The ethos of the course has been that the students should move away from a secondary school atmosphere towards becoming independent learners. However, we now realise that this is a new approach for most of the tutors. Several did not fully appreciate the need for a shift from a traditional, authoritative approach to a more facilitative one.

It also became apparent that the students did not know what was required of them in the open-ended section. Many of them did search the Web but felt that finding appropriate pages and downloading them was sufficient. In future there should be some discussion about how to go abut open-ended questions - how should the students go about deciding how much is needed etc.

Student Response to Project 2

Again the students in the focus group had enjoyed the project. They felt they were making connections between the project work and other subjects, particularly in terms of ways of tackling things. They did, however, feel there were some problems with the tutors not supporting their work or making time in the tutorials to discuss their findings. This is a real problem that needs to be addressed. Inevitably different tutors will have different approaches to tutorials, even within the defined ethos of the course. It should be reiterated that the students must have the space to use tutorials to discuss their work and gain confidence in talking about mathematics. This ability will be directly assessed through the oral exam and is an essential skill for engineers to acquire. The students also found difficulty in selecting components with relevant values – eg they were used to being told "you have a resistor of 10 ohms" rather than "what would be an appropriate size of resistor to make this circuit work?" Getting the students to select appropriate values and look at the validity of their assumptions is a necessary skill but one which is not seen as part of mathematics. However it fits well into optimisation and design problems.

Student Response to Project 3

Some students enjoyed the freedom to find their own data. One group decided for themselves that the data provided in the attachments was out-of-date (1994) and emailed an oil company to ask for more recent estimates! Another had found out about some of the recent developments in oil-production technology so that inaccessible oil-fields might soon be viable. This was not a part of the project and did not in itself contribute to the assessed work, but did stimulate them and motivate them to do the rest of the work.

Other students, however, felt that it wasn't 'real' engineering. This underlines the need to challenge their perceptions of what it means to be an engineer and what working on engineering problems as graduates will entail. To try and get them interested in a variety of problems. To relate to the real world and enjoy analysing the assumptions on which predictions are made. Many students did not understand what a model is – they did not realise how the same data could make different predictions about the future depending on the assumptions of the model. Again this is an important concept to inculcate at this time. Engineers must always understand the assumptions and validity of their models. By widening the relevance of the particular model or technique many will find the work more interesting.

Response to Project 4

This took place at the start of the third term and some students did not seem to spend as much time on it as in previous projects. Although at the oral some students said they had concentrated most on this one, having developed the requisite skills in earlier projects. It may be a good idea to change the pace and organisation – eg run it over 2 days, have a larger component of construction etc. Again we were trying to emphasise assumptions and increasing complexity of a model.

SO DOES THE MODEL WORK

An important question we need to ask is 'So does this way of *teaching* mathematics work?' For the answer is a qualified yes. This is not an unexpected answer, so single approach is going to yield a universal solution, different students respond to teaching methods in a variety of ways. In this study a focus group interview was conducted with a group of students at the end of each block to ascertain feedback on the delivery. It is interesting to report that views on the approach changed during the course of the year, at the start many students commented that they wanted to be told what to do in the project, that they were unsure of what was required in the open ended questions, that they were unsure how much staff wanted. As they progressed through the projects these concerns waned and by the end had disappeared. Indeed, many students commented favourably on the opportunity to work on 'real engineering problems in such an open ended manner'.

As a result of the first running of the course in academic session 1999 – 2000 feedback particularly on the oral assessment lead the course team to introduce a formative oral assessment after the second project. This was carried out in academic session 2000 – 2001 and feedback from students has been that this has been well received. At the time of writing it is not possible on comment on whether this has improved performance at the summative oral.

It is also fair to say that this method of teaching and learning mathematics has had benefits more widely. Students have developed their group wroking skills, their presentation skills, been introduced to mentoring and to peer support, the later two haveing been identified as helpful in improving retention rates, [5].

FURTHER ISSUES

Though this approach has been running for two academic sessions there are still issues that require attention. For example not all of those involved in the teaching are whole-heartedly committed to the model and this can prove problematic. Equally getting all of those involved in the setting of tests to set questions at a similar level, or those involved in the oral assessments to operate to the same methodology of conducting the oral is not without problems, but these are being worked on and we have confidence that as staff experience with this method grows then these issues will become less important.

What is clear is that this way of working does pick up students who are having difficulty early in the course and allows remedial action to be taken when it can still have an impact. Student do seem to enjoy working on the project, particularly where they are firmly anchored in electrical and electronic engineering.

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